MAT123 MATHEMATICS I

Lecture 03: Trigonometric Functions, Exponential Functions, Logarithmic Functions

Outline

The Trigonometric Functions

Inverse Functions

Exponential and Logarithmic Functions

The Trigonometric Functions

Trigonometric functions are fundamental in mathematics, particularly in geometry and calculus. They relate the angles of a triangle to the lengths of its sides.

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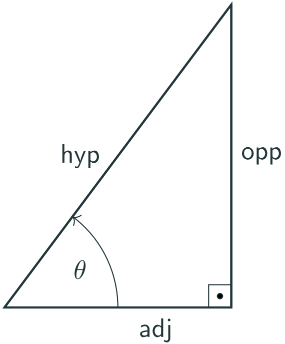


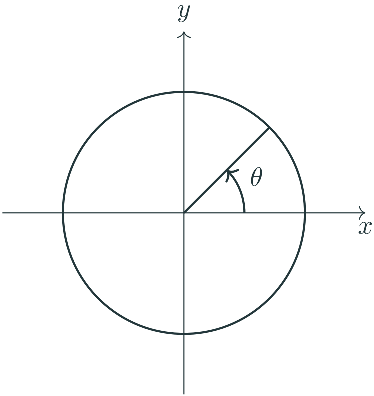
The three primary trigonometric functions are:

- Sine ($\sin$)
- Cosine ($\cos$)
- Tangent ($\tan$)

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}, \quad \cos(\theta) = \frac{\text{adj}}{\text{hyp}},$$
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

Unit Circle

The unit circle is a circle with a radius of 1 centered at the origin of the coordinate plane. The angle θ is measured from the positive x-axis.



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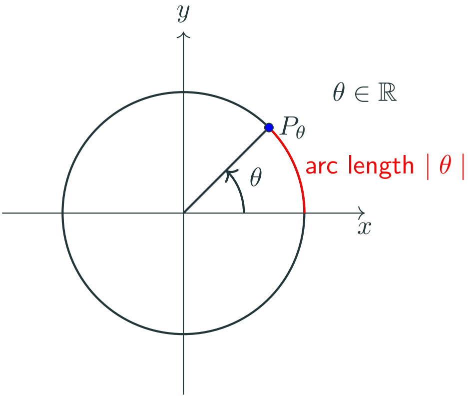

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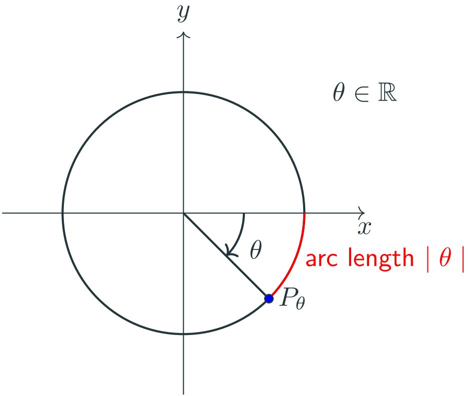

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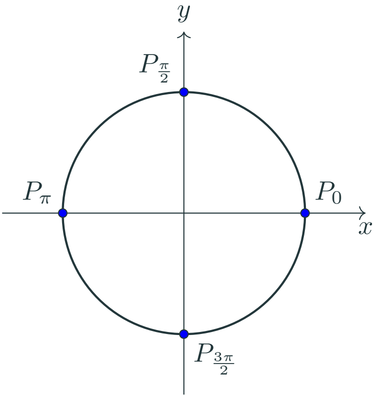

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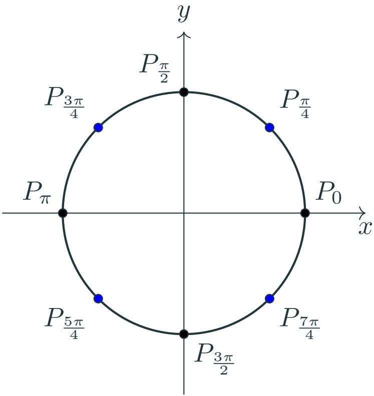

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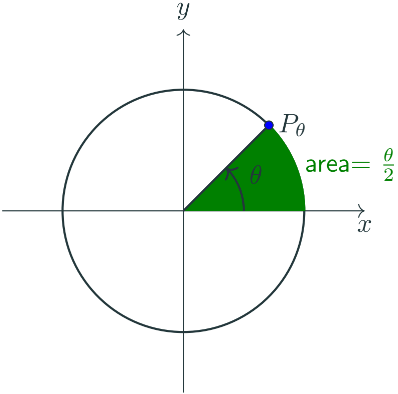
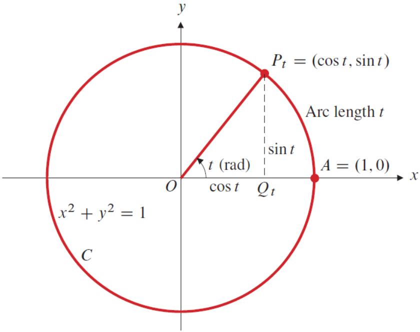


Given a real number t, the coordinates of the point P_t on the unit circle are given by:

$$P_t = (\cos(t), \sin(t)).$$



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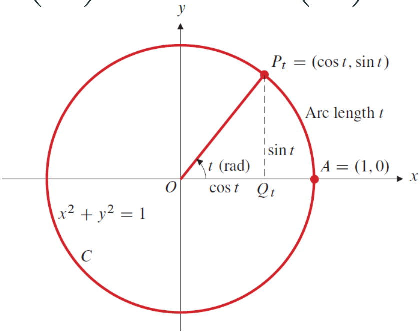
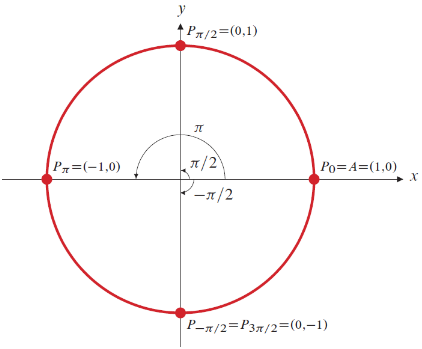
$$\sin^2(x) + \cos^2(x) = 1$$





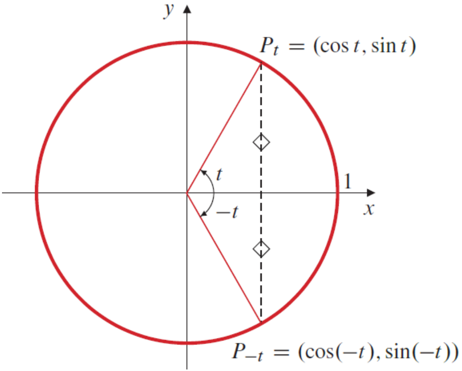
Special Angles

The values of sine and cosine for some special angles are:

$$\cos(0) = 1, \qquad \sin(0) = 0,$$
$$\cos\left(\frac{\pi}{2}\right) = 0, \qquad \sin\left(\frac{\pi}{2}\right) = 1,$$
$$\cos(\pi) = -1, \qquad \sin(\pi) = 0,$$
$$\cos\left(\frac{3\pi}{2}\right) = 0, \qquad \sin\left(\frac{3\pi}{2}\right) = -1.$$

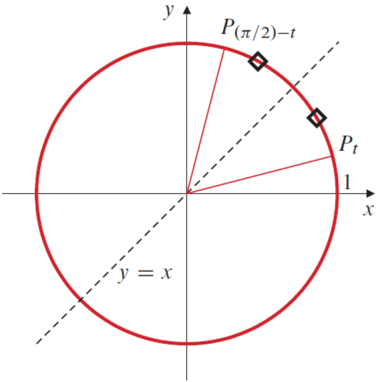
Cosine is an even function. Sine is an odd function.

- $\cos(-t) = \cos(t)$
- $\sin(-t) = -\sin(t)$



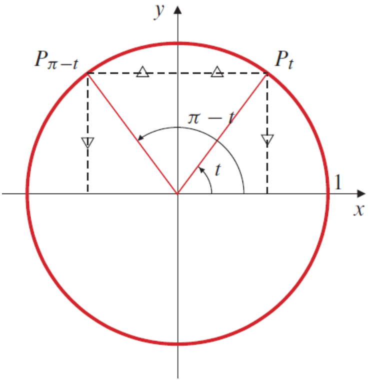
Complementary Angle Identities

- $\sin\left(\frac{\pi}{2} - t\right) = \cos(t)$
- $\cos\left(\frac{\pi}{2} - t\right) = \sin(t)$



Supplementary Angle Identities

- $\sin(\pi - t) = \sin(t)$
- $\cos(\pi - t) = -\cos(t)$

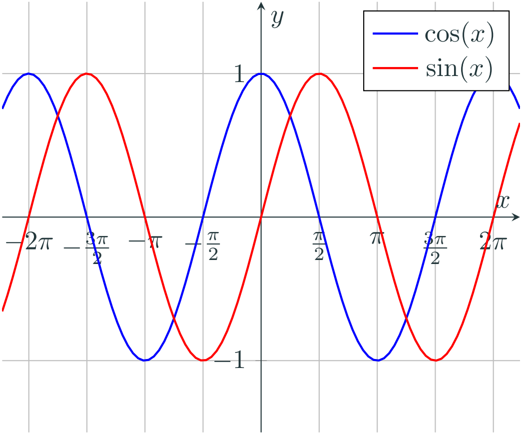


Cosines and Sines of Special Angles

Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

The Trigonometric Functions

Cosine and Sine



Addition Formulas

The addition formulas for sine and cosine are:

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

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Subtraction Formulas

The subtraction formulas are:

- $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
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Example

Find the value of $\cos(\pi/12)$.

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Example

Find the value of $\cos(\pi/12)$.

Solution

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}.$$

Double Angle Formulas

The double angle formulas for sine and cosine are:

- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

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Half Angle Formulas

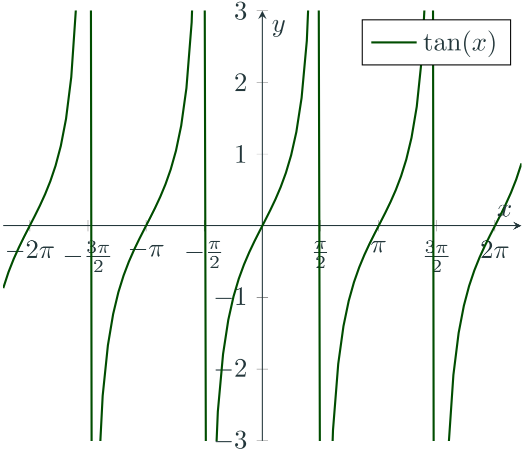
The half angle formulas are:

- $\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{2}$
- $\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos(x)}{2}$

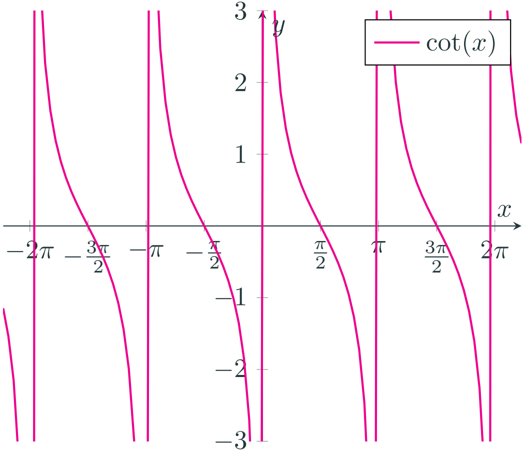
Tangent, cotangent, secant, and cosecant

In addition to sine and cosine, there are other trigonometric functions:

- Tangent: $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- Cotangent: $\cot(x) = \frac{\cos(x)}{\sin(x)}$
- Secant: $\sec(x) = \frac{1}{\cos(x)}$
- Cosecant: $\csc(x) = \frac{1}{\sin(x)}$



3
2
1
−1
−2
−3
y
x
tan(x)
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π



3
2
1
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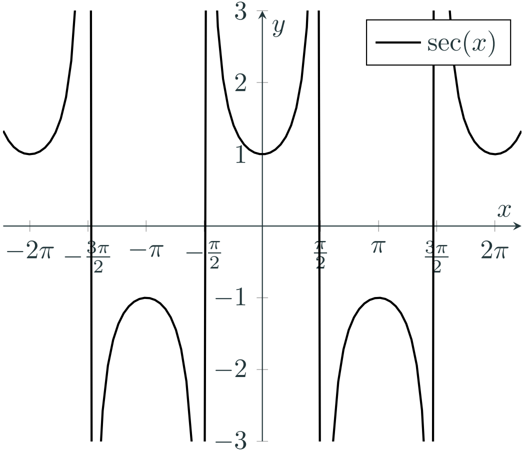
−2
−3
y
x
cot(x)
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π



sec(x)
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1
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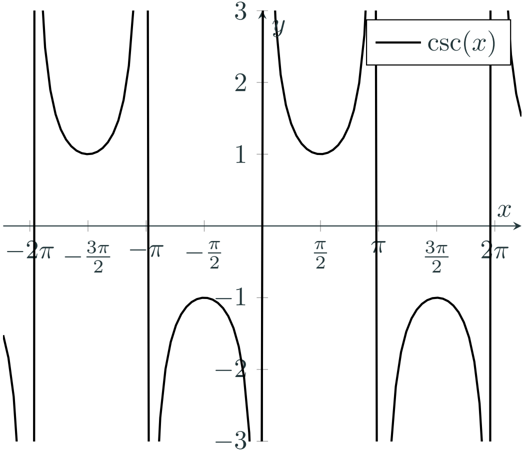
π/2
π
3π/2
2π

The Trigonometric Functions

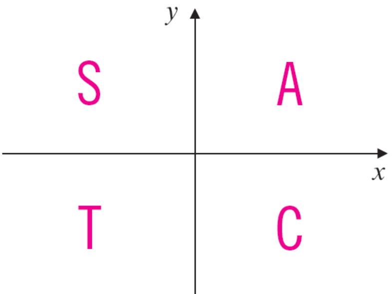
Other Trigonometric Functions



CAST Rule

The CAST rule helps to determine the signs of trigonometric functions in different quadrants:

- Quadrant I: All functions are positive.
- Quadrant II: Sine and cosecant are positive.
- Quadrant III: Tangent and cotangent are positive.
- Quadrant IV: Cosine and secant are positive.



The Trigonometric Functions

Example

Find the sine and tangent of the angle θ in $\left[\pi, \frac{3\pi}{2}\right]$ for which $\cos(\theta) = -\frac{1}{3}$.

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Solution

Since θ is in the third quadrant, we have:

$$\sin(\theta) = -\sqrt{1-\cos^2(\theta)} = -\sqrt{1-\left(-\frac{1}{3}\right)^2} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}.$$

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The tangent is given by:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = 2\sqrt{2}.$$

Other Trigonometric Identities

- $1 + \tan^2(x) = \sec^2(x)$

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- $\tan(x \mp y) = \dfrac{\tan(x) \mp \tan(y)}{1 \pm \tan(x)\tan(y)}$

Sine and Cosine Laws

- Law of Sines:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

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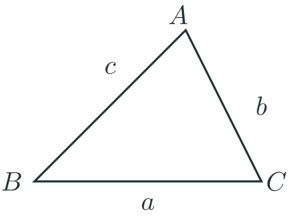
- Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$



Inverse Functions

Inverse Functions

Definition *(One-to-One Function)*

A function $f : A \to B$ is called **one-to-one** (or injective) if it never assigns the same value in B to two different elements of A. Formally, if

$$f(x_1) = f(x_2) \implies x_1 = x_2$$

for all $x_1, x_2 \in A$, then f is one-to-one.

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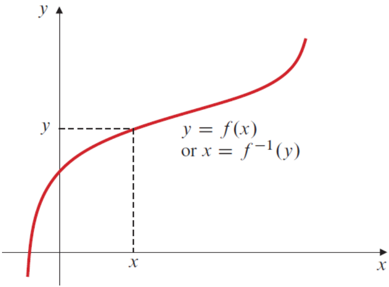

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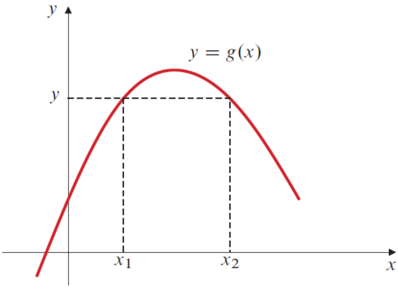

not one-to-one

Inverse Functions

Definition *(Inverse Function)*

If the function $f : A \to B$ is one-to-one, then it has an **inverse function** f^{-1} : range$(f) \to A$. The value of $f^{-1}(x)$ is the unique $y \in A$ such that $f(y) = x$.

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The inverse function f^{-1} "undoes" the action of f:

$$f(f^{-1}(y)) = y \quad \text{and} \quad f^{-1}(f(x)) = x$$

Inverse Functions

Finding Inverse Functions

To find the inverse of a function f:

1. Replace $f(x)$ with y.
2. Solve for x in terms of y.
3. Swap x and y.
4. Write the result as $f^{-1}(x) = \cdots$.

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Example

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Example

Find the inverse of the function $f(x) = 2x + 3$.

Solution

1. *Replace $f(x)$ with y:* $y = 2x + 3$.
2. *Solve for x:* $x = \frac{y-3}{2}$.
3. *Swap x and y:* $y = \frac{x-3}{2}$.
4. *Write the result:* $f^{-1}(x) = \frac{x-3}{2}$.

Inverse Functions

Properties of Inverse Functions

- The domain of f is the range of f^{-1} and vice versa.

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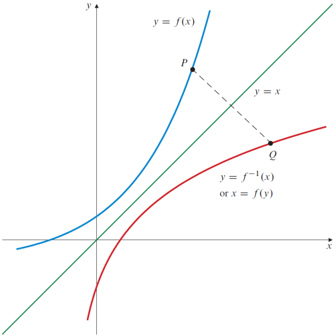
- $(f^{-1})^{-1} = f$
- The graph of f^{-1} is the reflection of the graph of f across the line $y = x$.



Example

Show that the function

$$f(x) = \frac{1-2x}{1+x}$$

is one-to-one, and calculate its inverse. Specify the domain and range of both f and f^{-1}.

Solution

Inverse Functions

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Solution

- To show that f is one-to-one, we can use the horizontal line test or show that $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

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Solution

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- The inverse can be found by solving for x:

$$y = \frac{1-2x}{1+x} \implies y(1+x) = 1-2x \implies y + yx = 1-2x.$$

Example

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Solution

- Rearranging gives:

$$(y+2)x = 1-y \implies x = \frac{1-y}{y+2}.$$

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- Thus, the inverse is:

$$f^{-1}(y) = \frac{1-y}{y+2}.$$

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Inverse Functions

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is one-to-one, and calculate its inverse. Specify the domain and range of both f and f^{-1}.

Solution

- Thus, we have:

$$\text{Domain of } f : (-\infty, -1) \cup (-1, \infty)$$

$$\text{Range of } f : (-\infty, -2) \cup (-2, \infty)$$

$$\text{Domain of } f^{-1} : (-\infty, -2) \cup (-2, \infty)$$

$$\text{Range of } f^{-1} : (-\infty, -1) \cup (-1, \infty)$$

Exponential and Logarithmic Functions

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$$f(x) = a^x$$

where $a > 0$ and $a \neq 1$. The base a is a constant, and the variable x is the exponent.

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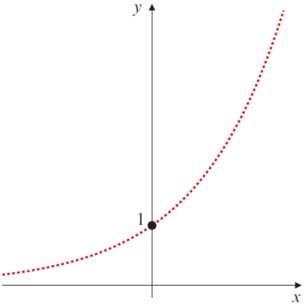

$y = 2^x$ for rationals

An Exponential function is a function of the form:

$$f(x) = a^x$$

where $a > 0$ and $a \neq 1$. The base a is a constant, and the variable x is the exponent.

How can we raise a number to an irrational exponent?

$y = 2^x$ for rationals

- If x is a positive integer, then a^x is the product of x factors of a.

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- For rational numbers, we define:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Irrational powers can be approximated by rational powers.

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Example

Let's find the value of $2^{\sqrt{3}}$.

$$\begin{aligned}
1.7 < \sqrt{3} < 1.8 &\implies 2^{1.7} < 2^{\sqrt{3}} < 2^{1.8} \\
1.73 < \sqrt{3} < 1.74 &\implies 2^{1.73} < 2^{\sqrt{3}} < 2^{1.74} \\
1.732 < \sqrt{3} < 1.733 &\implies 2^{1.732} < 2^{\sqrt{3}} < 2^{1.733} \\
1.7320 < \sqrt{3} < 1.7321 &\implies 2^{1.7320} < 2^{\sqrt{3}} < 2^{1.7321} \\
1.73205 < \sqrt{3} < 1.73206 &\implies 2^{1.73205} < 2^{\sqrt{3}} < 2^{1.73206} \\
\vdots \qquad & \qquad \vdots
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\vdots & & \rightarrow \vdots \leftarrow
\end{array}$$

There is exactly one number greater than all the numbers in the yellow rectangle and less than all the numbers in the red rectangle. This number is $2^{\sqrt{3}}$.

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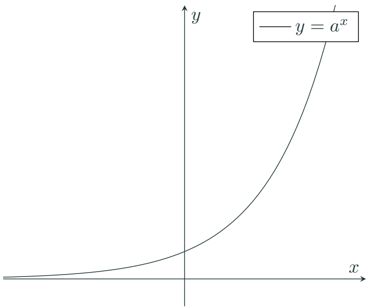
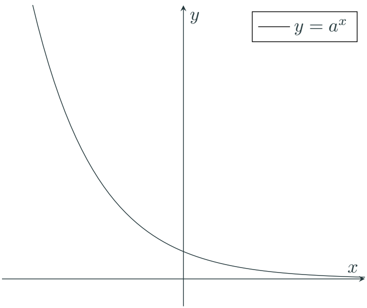
Properties of Exponential Functions

- The domain is all real numbers. The range is $(0, \infty)$.
- $a^0 = 1$
- $a^{-x} = \frac{1}{a^x}$
- $a^{x+y} = a^x \cdot a^y$
- $a^{x-y} = \frac{a^x}{a^y}$
- $a^{xy} = (a^x)^y$
- $(ab)^x = a^x \cdot b^x$



$a > 1$



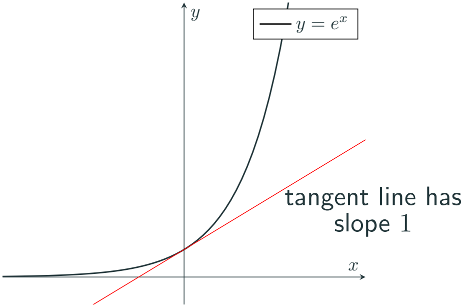
$0 < a < 1$

Natural Exponential Function

The natural exponential function is defined as:

$$f(x) = e^x$$

where e is Euler's number, approximately 2.71828.



Logarithmic Function

The logarithmic function is defined as the inverse of the exponential function:

$$f(x) = \log_a(x)$$

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- $\log_a(x^r) = r \log_a(x)$

Logarithmic Function

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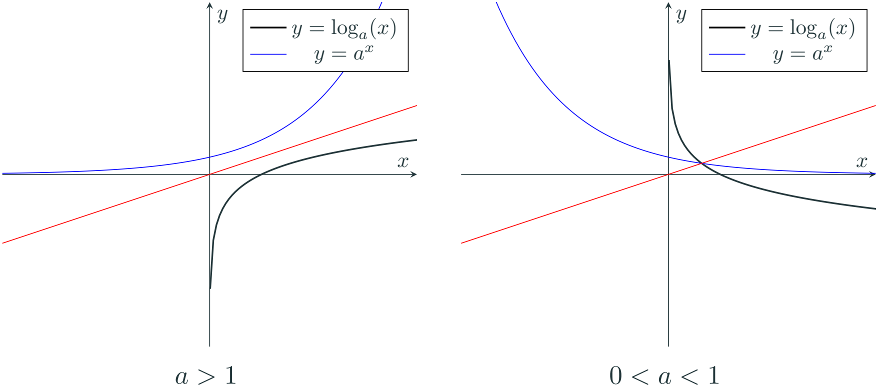
Properties of Logarithmic Functions

- $\log_a(a^x) = x$ for all $x \in \mathbb{R}$
- $a^{\log_a(x)} = x$ for all $x > 0$



y
x
$y = \log_a(x)$
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y
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Example

Solve the equation $3^{x-1} = 2^x$.

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- Use the properties of logarithms:

$$(x-1)\log(3) = x\log(2)$$

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- Factor out x:

$$x(\log(3) - \log(2)) = \log(3)$$

Example

Solve the equation $3^{x-1} = 2^x$.

Solution

The solution is:

$$x = \frac{\log(3)}{\log(3) - \log(2)} = \frac{\log(3)}{\log\left(\frac{3}{2}\right)} = \log_{3/2}(3).$$