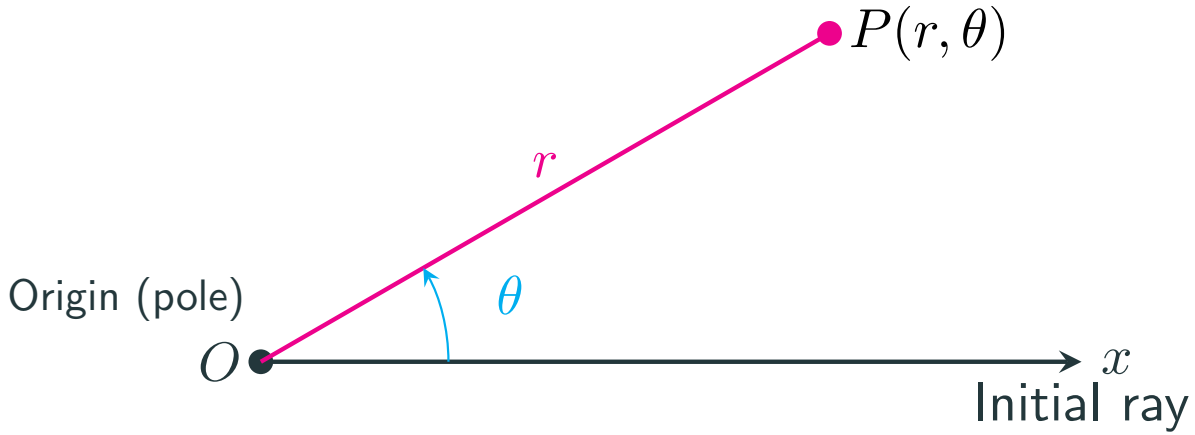
MAT124 MATHEMATICS II

Parametric Curves

Outline

Polar Coordinates

Polar Coordinates and Polar Curves



To define polar coordinates for the plane, we start with an origin, called the **pole**, and an **initial ray**.

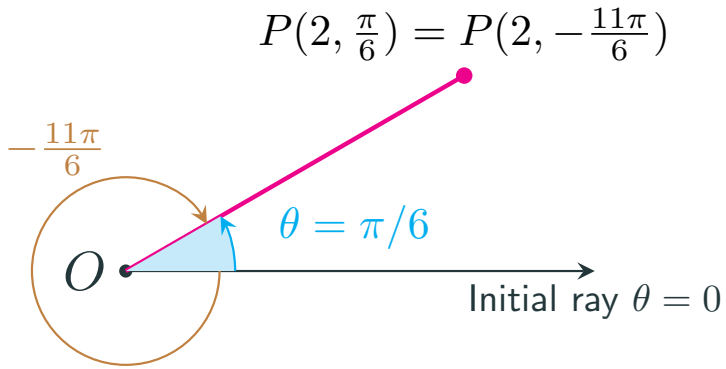
Polar Coordinates

$P(r, \theta)$

Directed distance from O to P

Directed angle from initial ray to OP

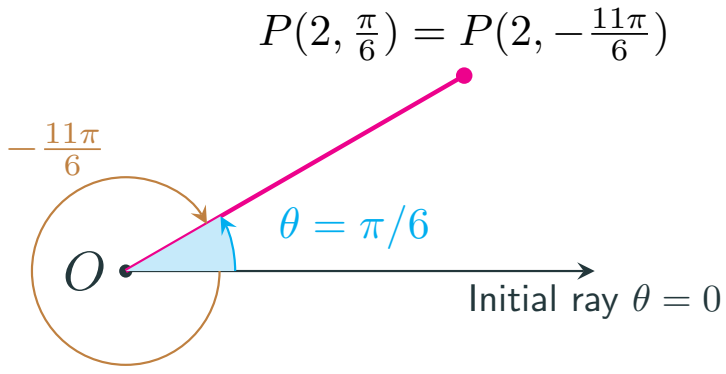
Coordinates are not unique



Polar coordinates of a point are **not unique**.

Polar Coordinates and Polar Curves

Coordinates are not unique



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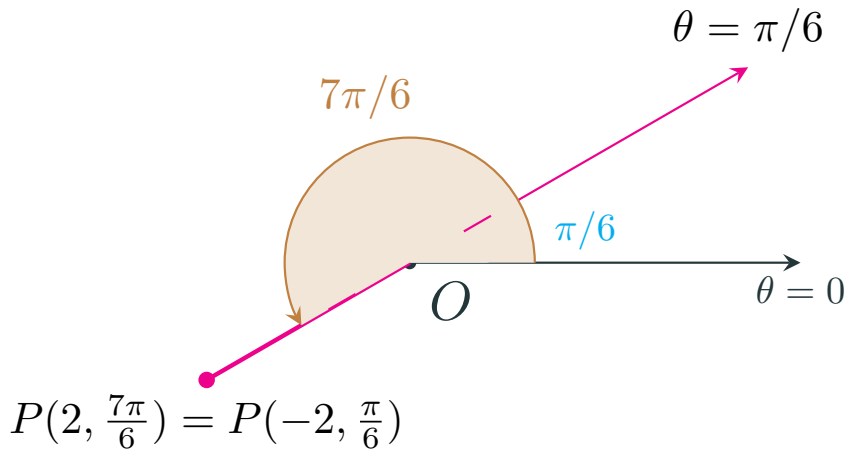
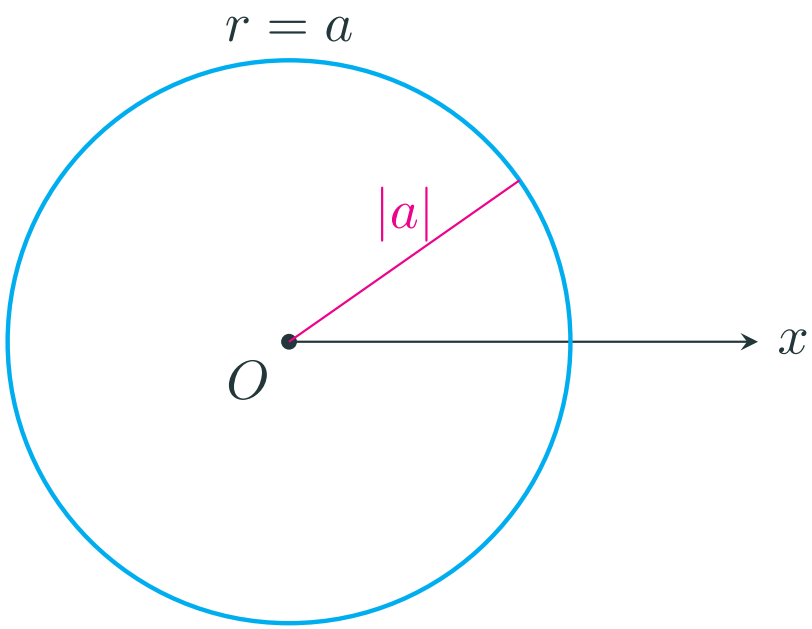
Negative r-values



Polar coordinates can have **negative r-values**.

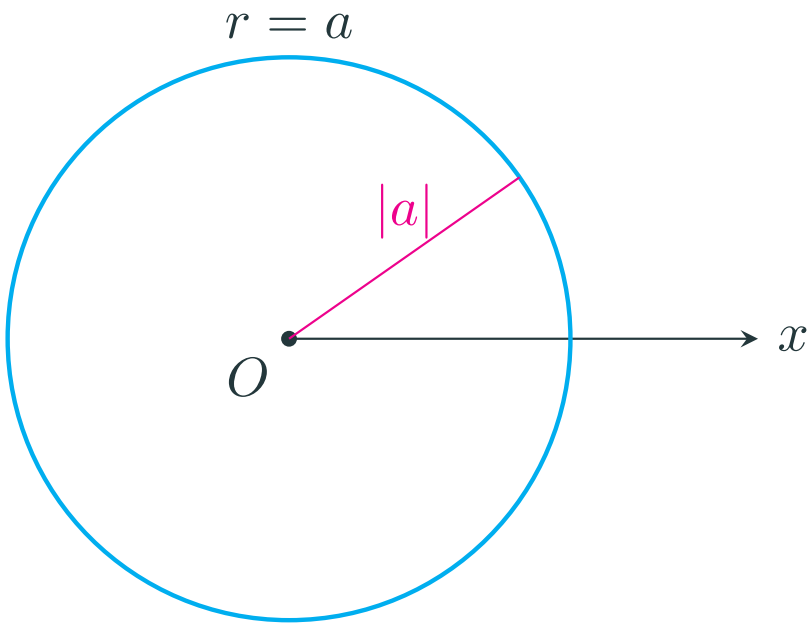
Polar Coordinates and Polar Curves

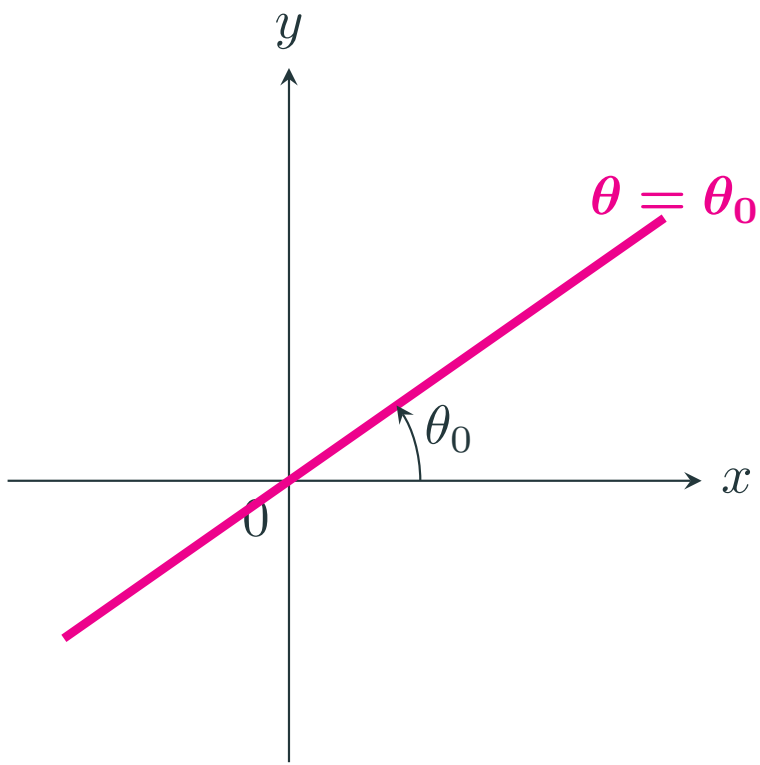
Equation	**Graph**
$r = a$	Circle of radius $\lvert a \rvert$ centered at O



Polar Coordinates and Polar Curves

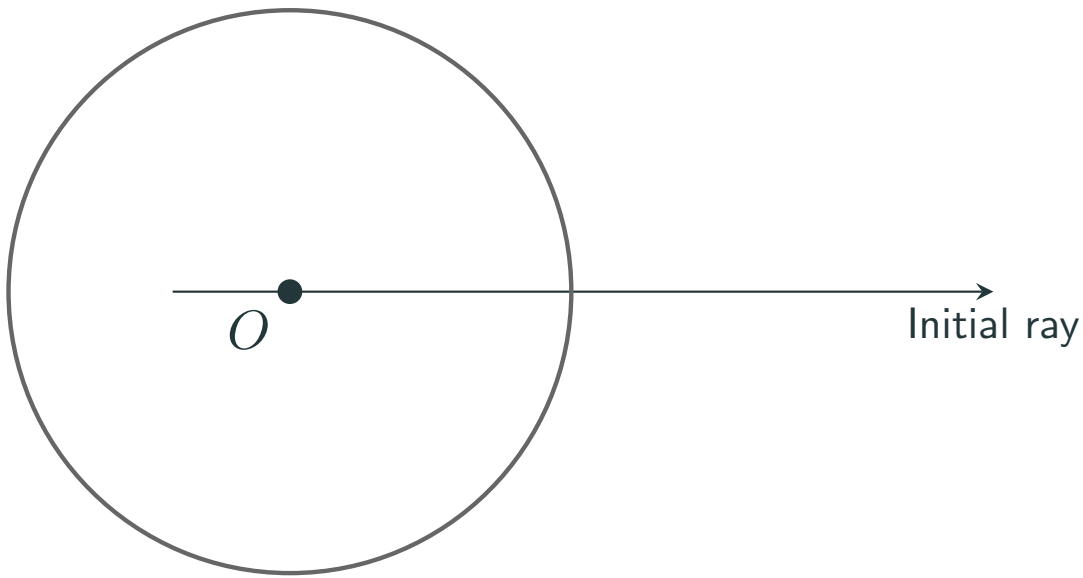
Equation	Graph
$r = a$	Circle of radius $\lvert a \rvert$ centered at O
$\theta = \theta_0$	Line through O making an angle θ_0 with the initial ray





Equations for Circles and Lines

(a) $r = 1$ and $r = -1$ are equations for the circle of radius 1 centered at O.



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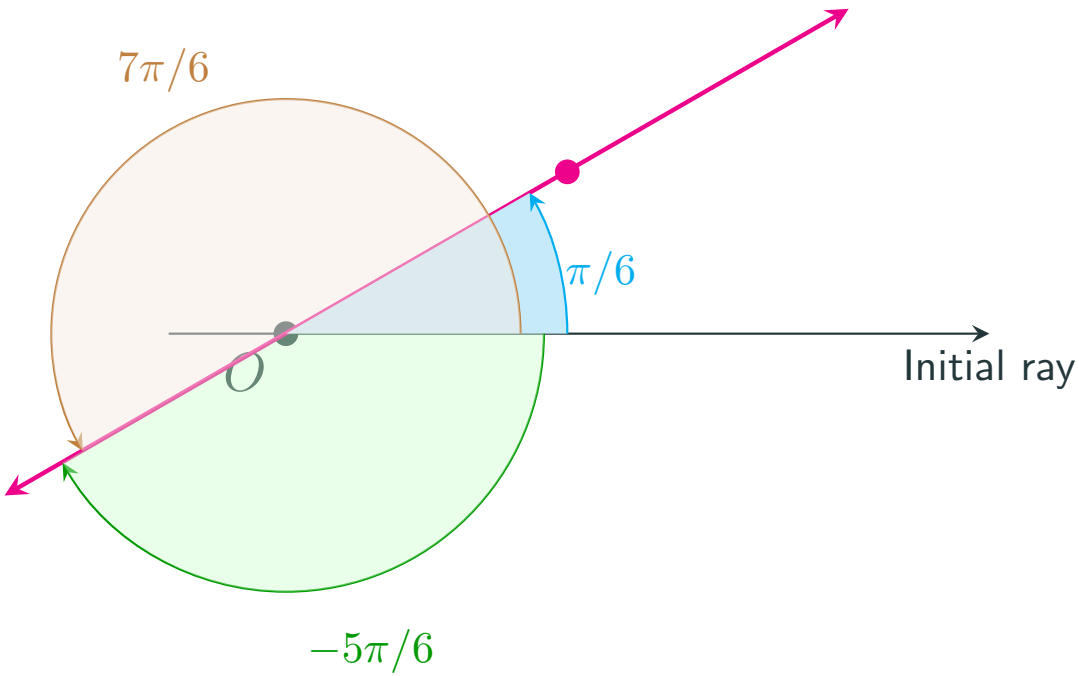
(a) $r = 1$ and $r = -1$ are equations for the circle of radius 1 centered at O.

(b) $\theta = \pi/6, \theta = 7\pi/6$, and $\theta = -5\pi/6$ are equations for the line shown below.



Polar Coordinates and Graphs

EXAMPLE: Graph the sets of points whose polar coordinates satisfy the following conditions:

(a) $1 \leq r \leq 2$ and $0 \leq \theta \leq \frac{\pi}{2}$

(b) $-3 \leq r \leq 2$ and $\theta = \frac{\pi}{4}$

(c) $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$ (no restriction on r)

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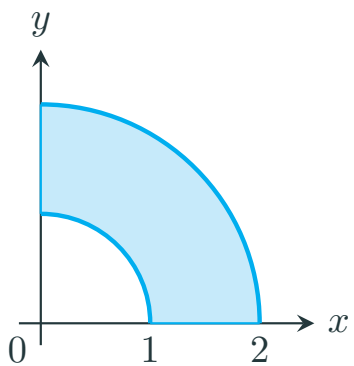

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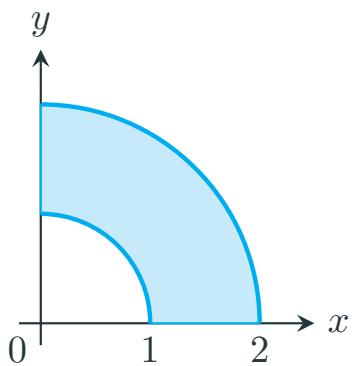

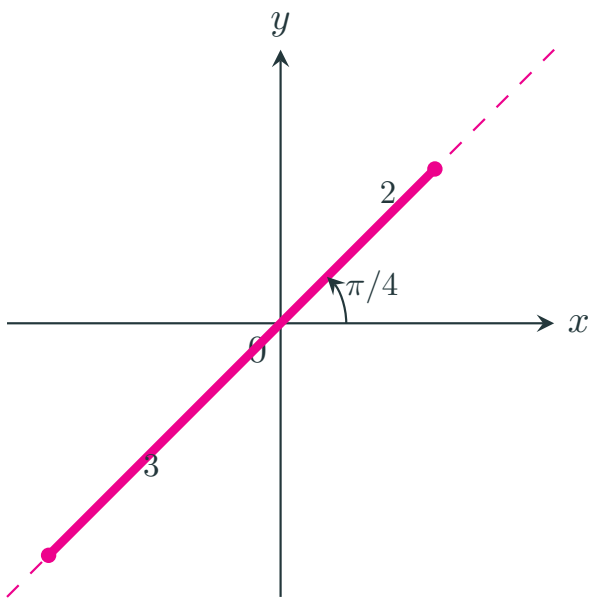

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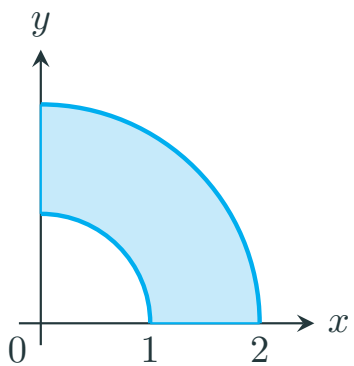

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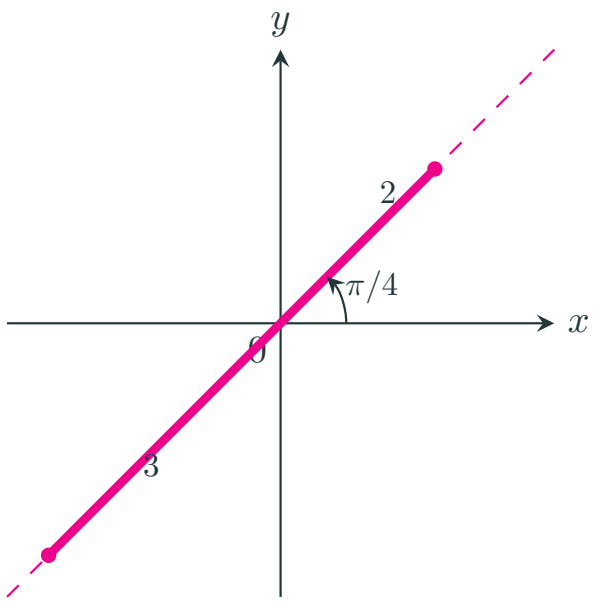

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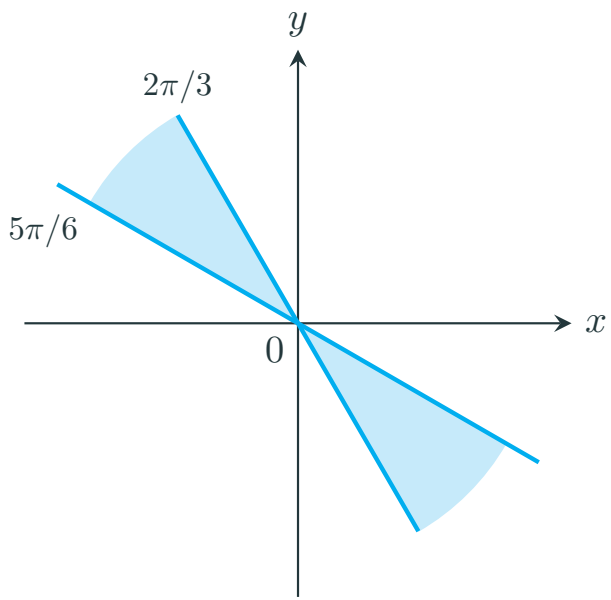
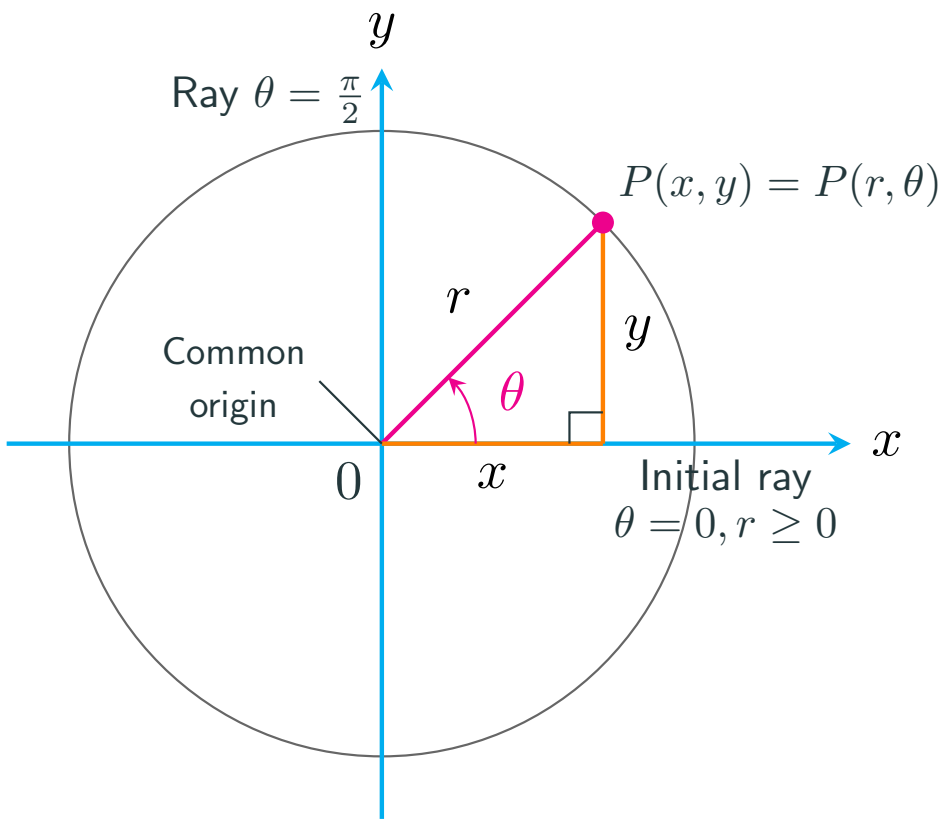


Equations Relating Polar and Cartesian Coordinates

$$x = r\cos\theta, \quad y = r\sin\theta, \quad r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$$



Converting Polar Equations to Cartesian

EXAMPLE: Find Cartesian equations for the following polar equations and describe their graphs.

(a) $r\cos\theta = 2$

(b) $r^2\cos\theta\sin\theta = 4$

(c) $r^2\cos^2\theta - r^2\sin^2\theta = 1$

(d) $r = 1 + 2r\cos\theta$

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Graph: A vertical line.

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Since $x = r\cos\theta$ and $y = r\sin\theta$, we have: $(r\cos\theta)(r\sin\theta) = 4$

This gives: $xy = 4$

Graph: A hyperbola.

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(d) $r = 1 + 2r\cos\theta$

$r = 1 + 2x \implies r^2 = (1+2x)^2$

$x^2 + y^2 = 1 + 4x + 4x^2 \implies 3x^2 - y^2 + 4x + 1 = 0$

Graph: A hyperbola.

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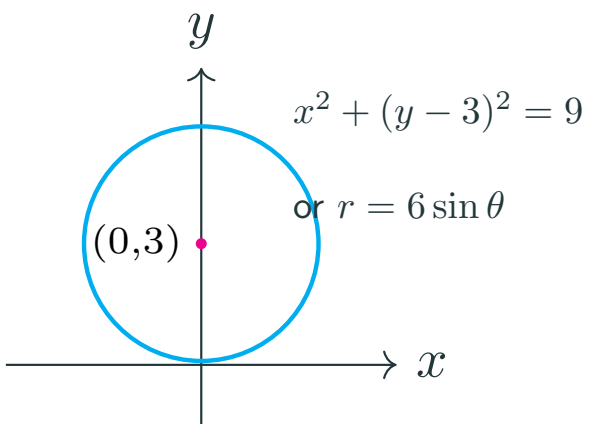

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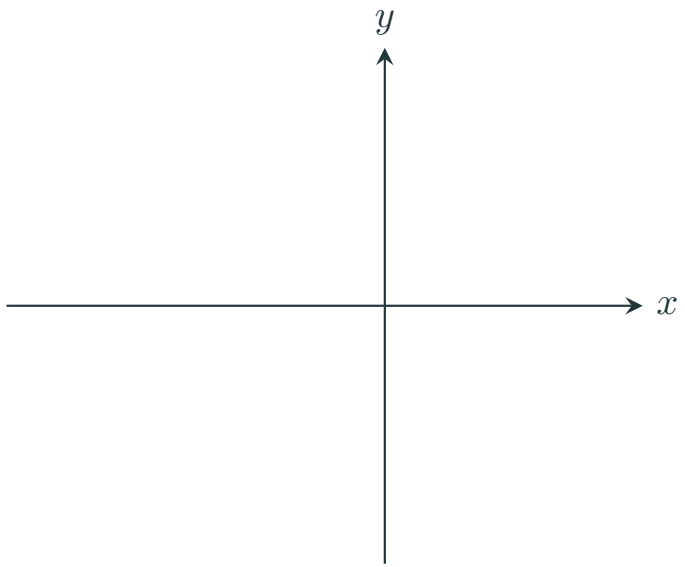

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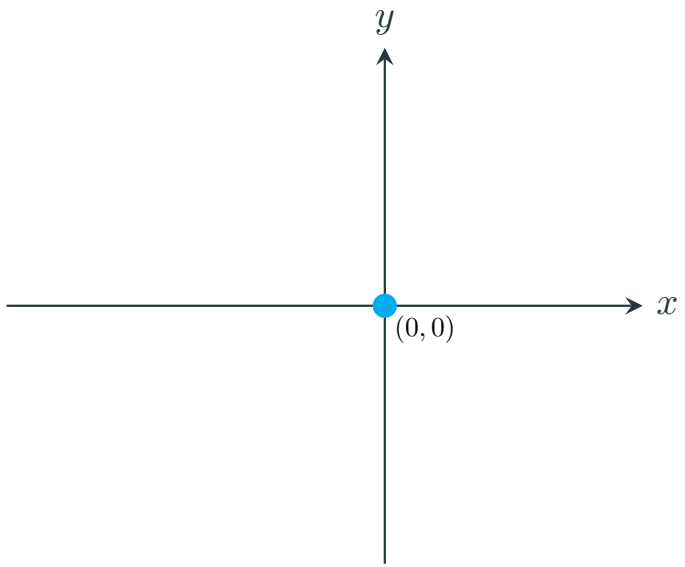
θ	$r = 1 - \cos\theta$
0	0



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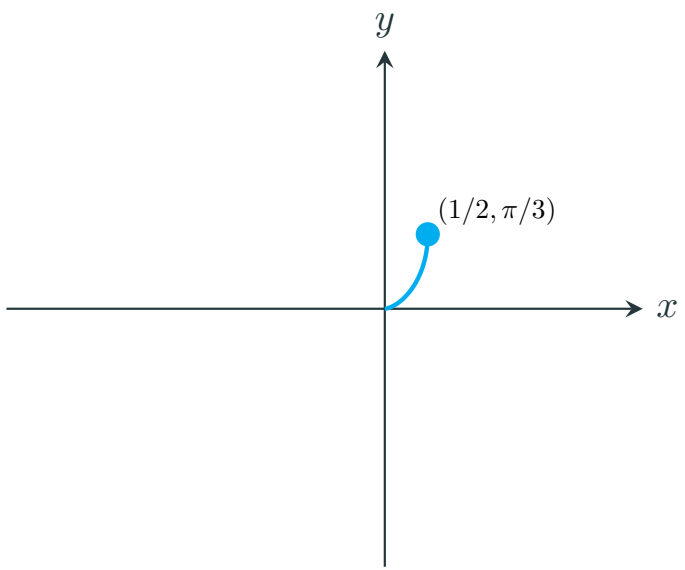
0	0
$\pi/3$	$1/2$



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0	0
$\pi/3$	$1/2$
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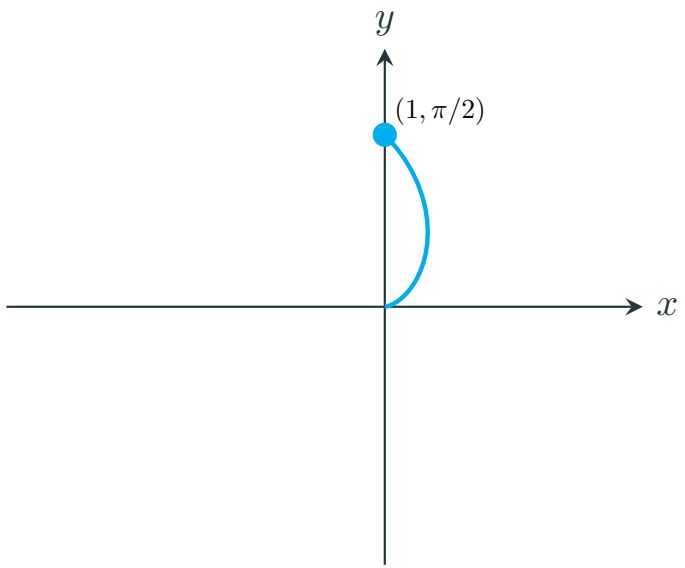

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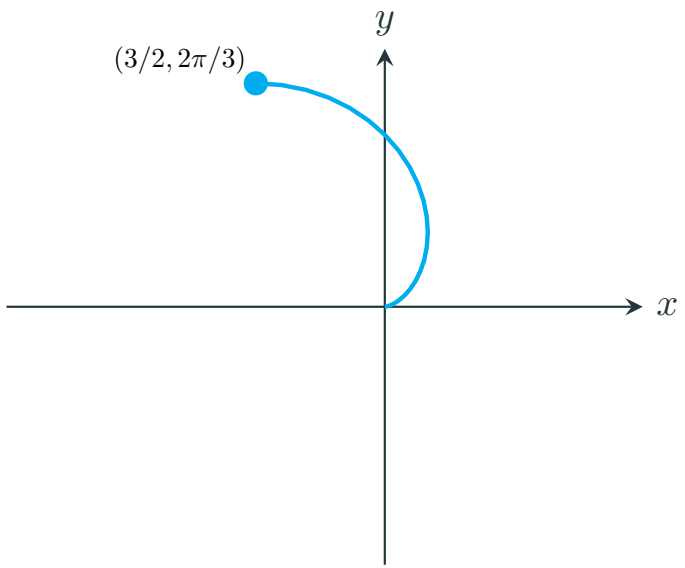

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0	0
$\pi/3$	$1/2$
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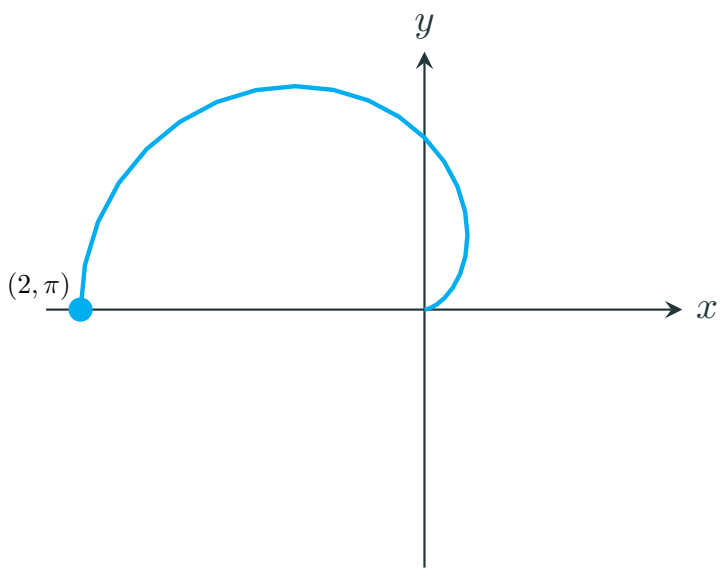
π	2



Final Step
Use symmetry to complete the lower half (180° to 360°).

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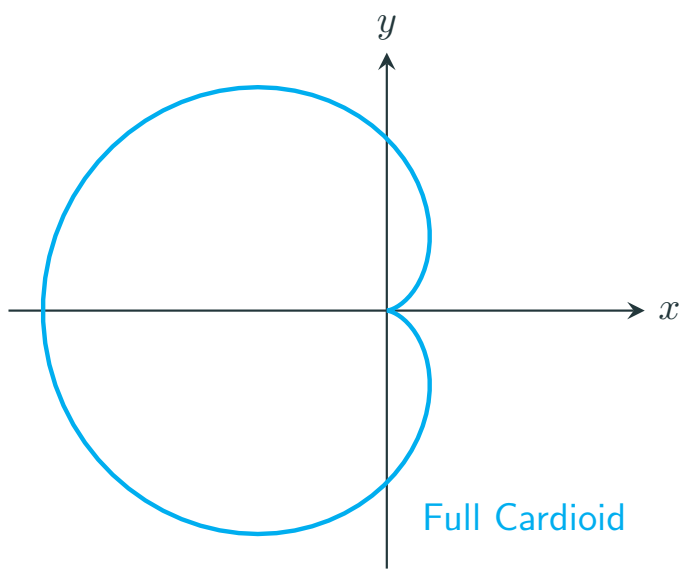

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The curve is **symmetric about the x-axis** because:

$$(r,\theta) \text{ on the graph} \implies r^2 = 4\cos\theta \implies r^2 = 4\cos(-\theta)$$
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The curve passes through the origin when $\theta = -\pi/2$ and $\theta = \pi/2$. It has a vertical tangent both times because $\tan\theta$ is infinite.

Graphing Polar Equations

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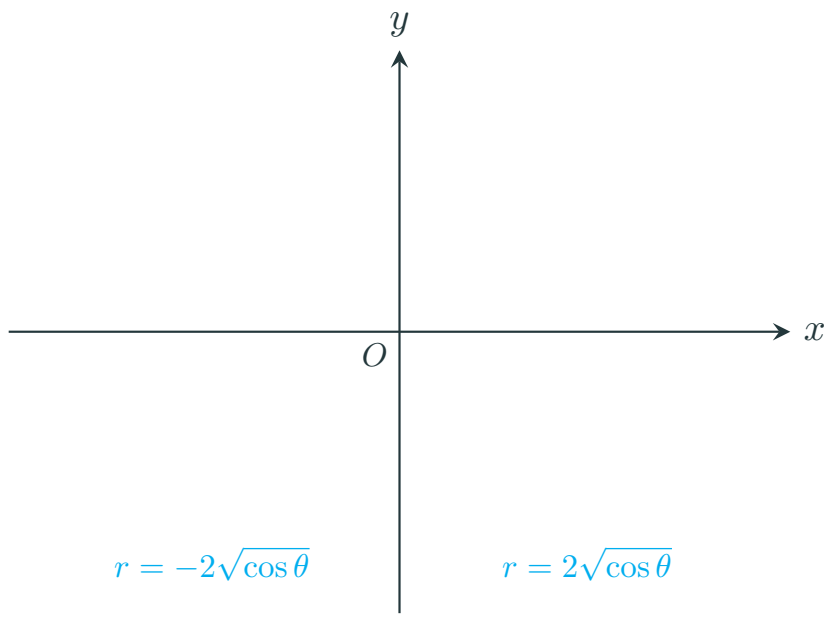
Graph the curve $r^2 = 4\cos\theta$.

Solution:

Curve: $r^2 = 4\cos\theta \implies r = \pm 2\sqrt{\cos\theta}$

For each $\theta \in [-\pi/2, \pi/2]$, the formula gives two values of r.

θ	$\cos\theta$	$r = \pm 2\sqrt{\cos\theta}$



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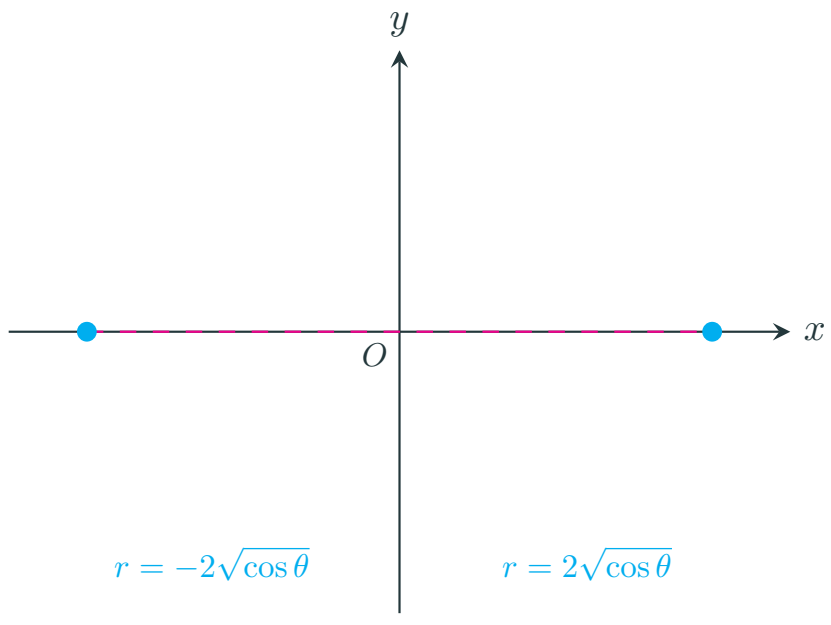
θ	$\cos\theta$	$r = \pm 2\sqrt{\cos\theta}$
0	1	± 2



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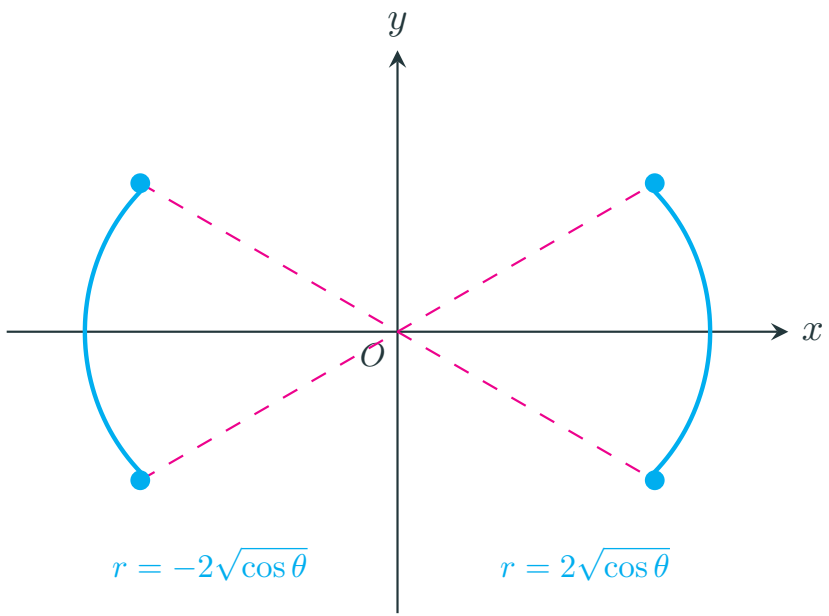
θ	$\cos\theta$	$r = \pm 2\sqrt{\cos\theta}$
0	1	± 2
$\pm\pi/6$	$\sqrt{3}/2$	$\approx \pm 1.9$



Graphing Polar Equations

EXAMPLE:
Graph the curve $r^2 = 4\cos\theta$.

Solution:

Curve: $r^2 = 4\cos\theta \implies r = \pm 2\sqrt{\cos\theta}$

For each $\theta \in [-\pi/2, \pi/2]$, the formula gives two values of r.

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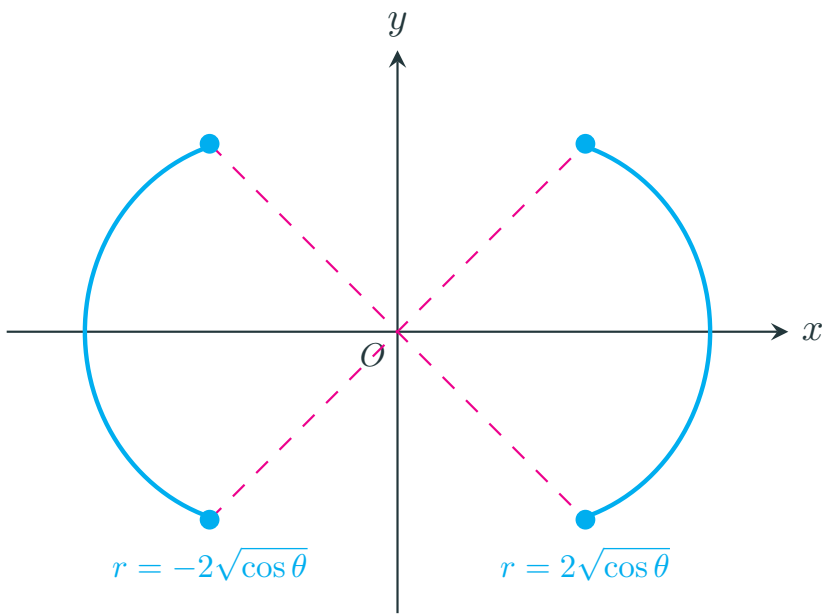
0	1	± 2
$\pm\pi/6$	$\sqrt{3}/2$	$\approx \pm 1.9$
$\pm\pi/4$	$1/\sqrt{2}$	$\approx \pm 1.7$



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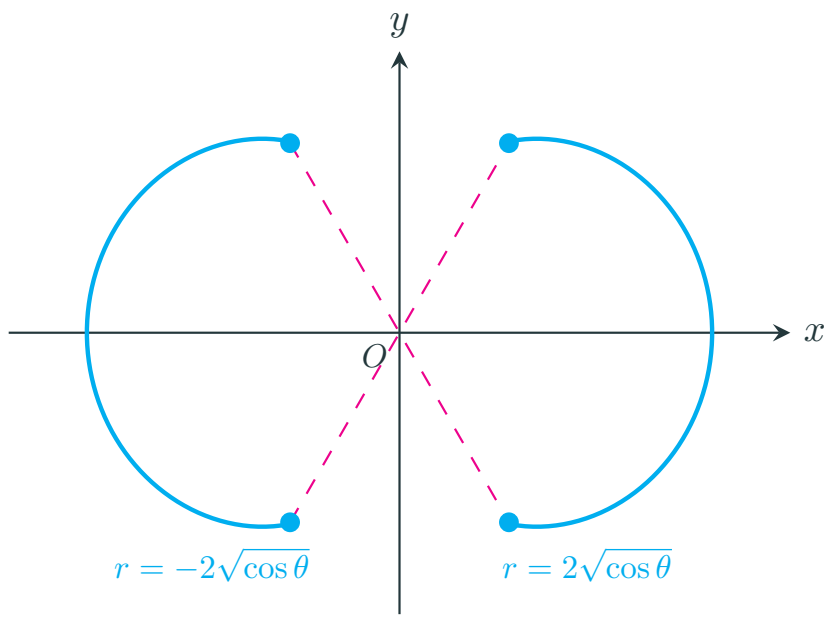

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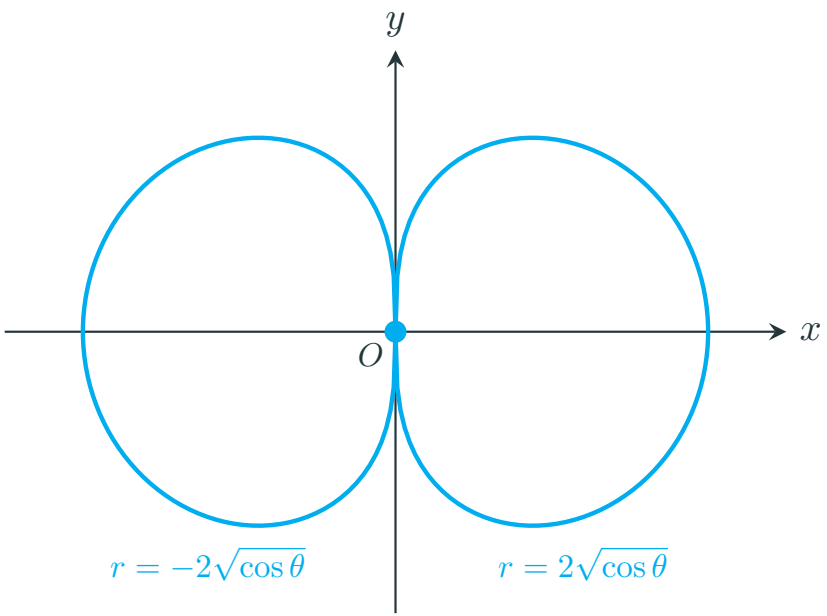

Graphing Polar Equations

A Technique for Graphing

One way to graph a polar equation $r = f(\theta)$ is to make a table of (r, θ)-values, plot the corresponding points, and connect them in order of increasing θ.

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This method works well if enough points are plotted to reveal all the loops and dimples in the graph.

However, it can be difficult to know how many points to plot and where to plot them.

Another method of graphing that is usually quicker and more reliable is to:

1. first graph $r = f(\theta)$ in the *Cartesian $r\theta$-plane*,
2. then use the Cartesian graph as a "table" and guide to sketch the *polar coordinate graph*.

Graphing Polar Equations

EXAMPLE:

Graph the lemniscate curve $r^2 = \sin 2\theta$.

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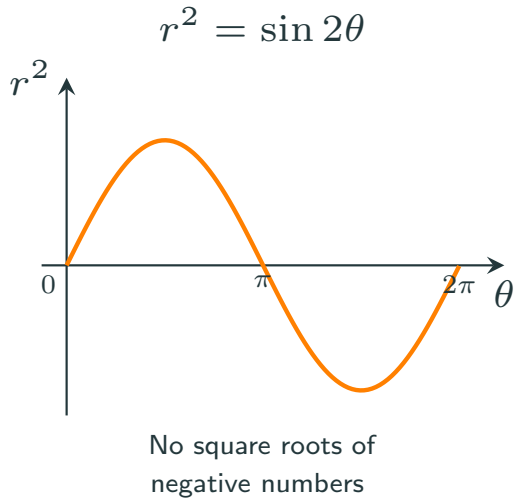

No square roots of
negative numbers

Graphing Polar Equations

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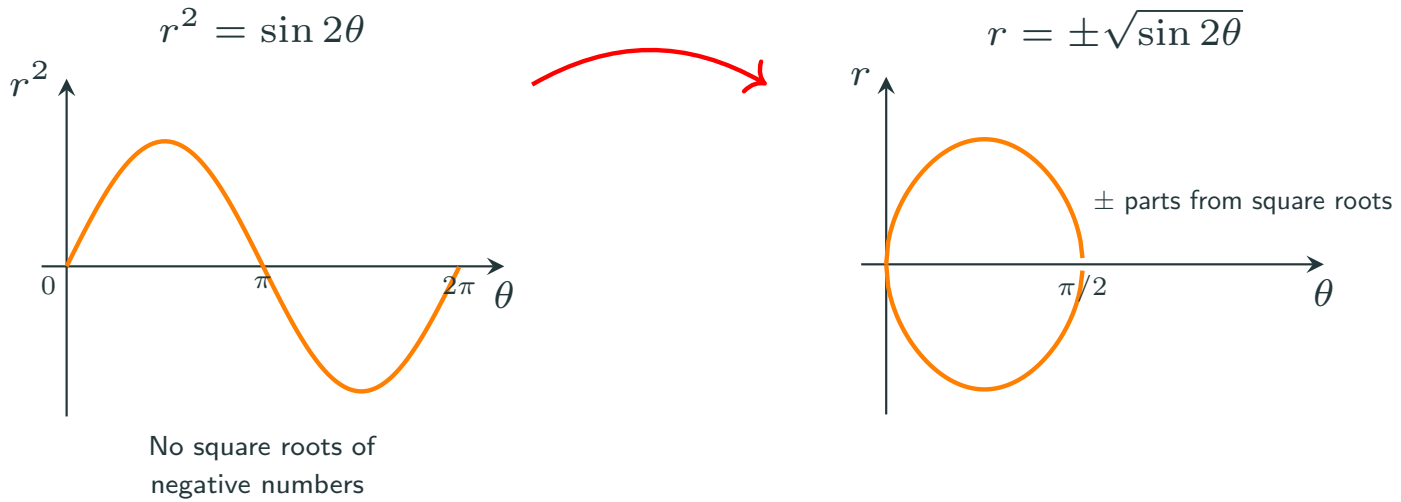

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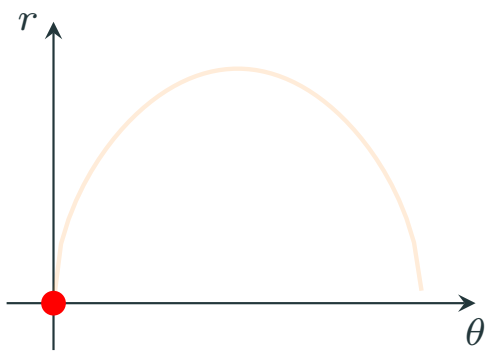
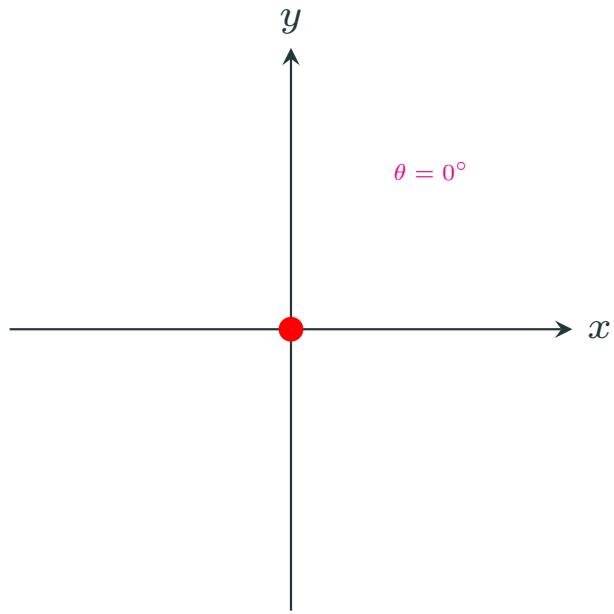
$r = \sqrt{\sin 2\theta}$ (Cartesian guide)



Corresponding Polar Graph $r^2 = \sin 2\theta$



- $\theta = 0 \to 45°$: r value increases from 0 to 1.

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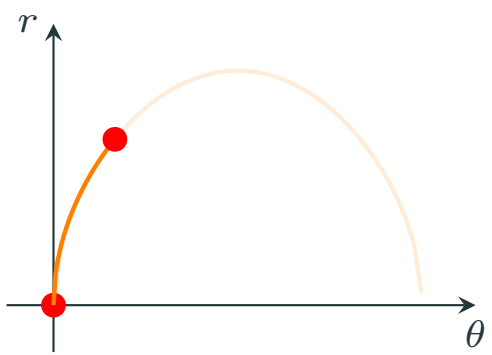

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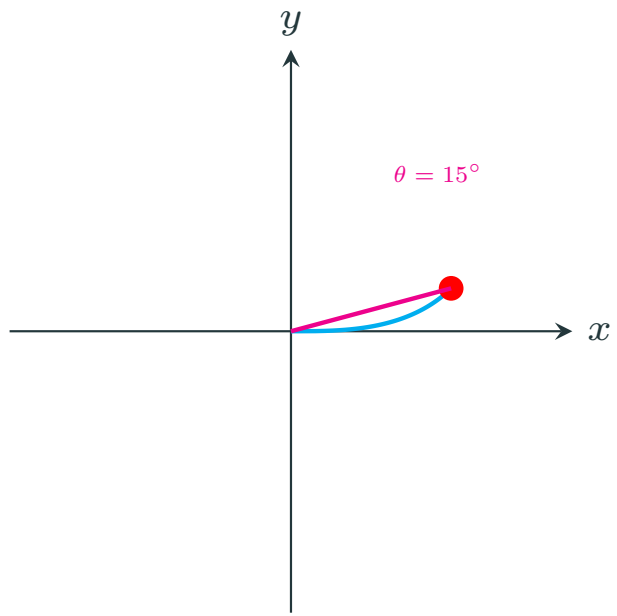

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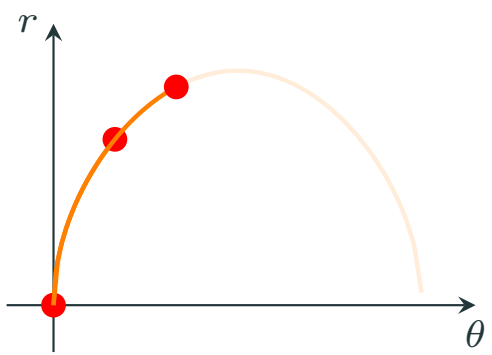

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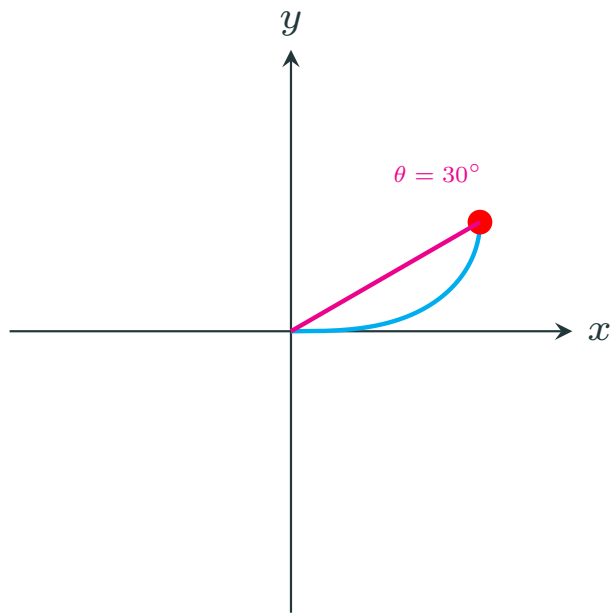

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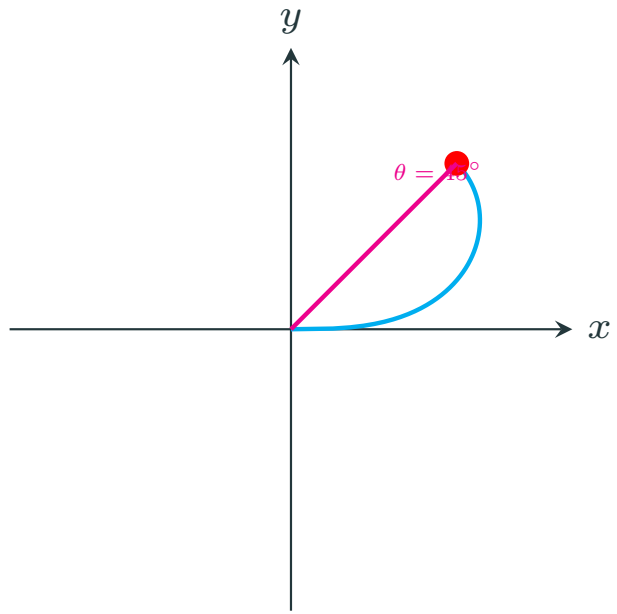

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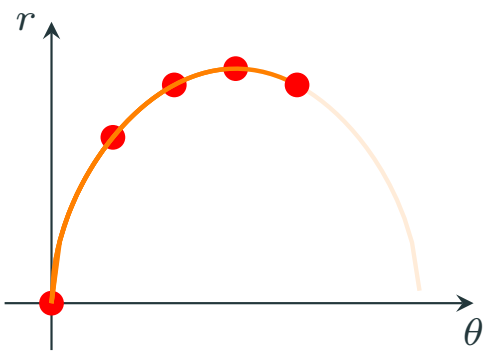

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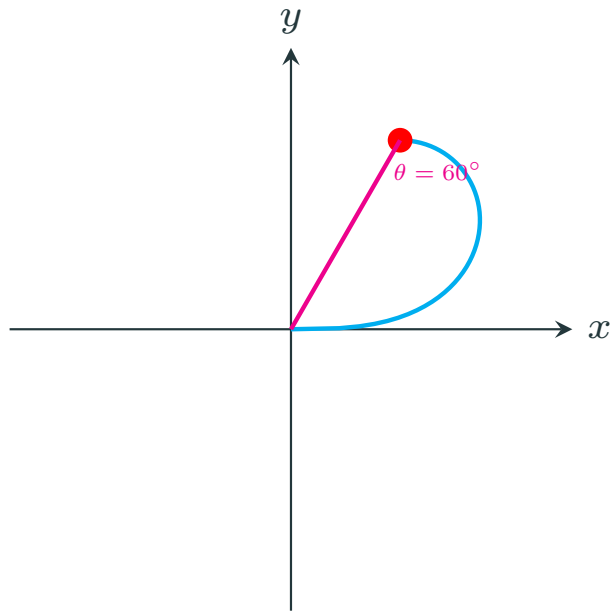

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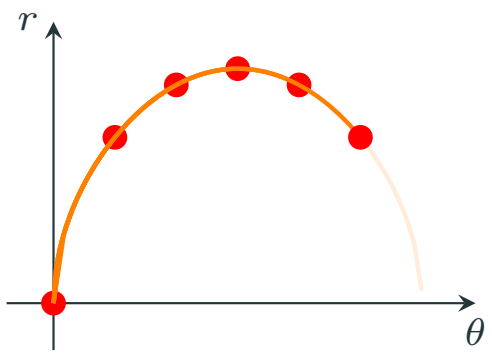

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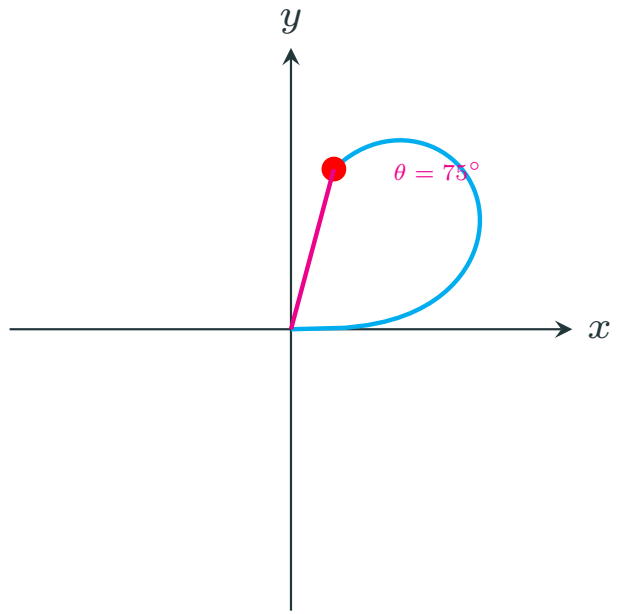

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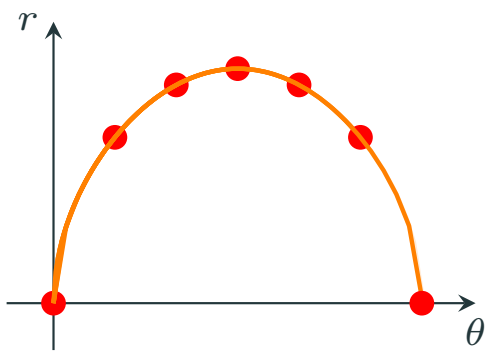

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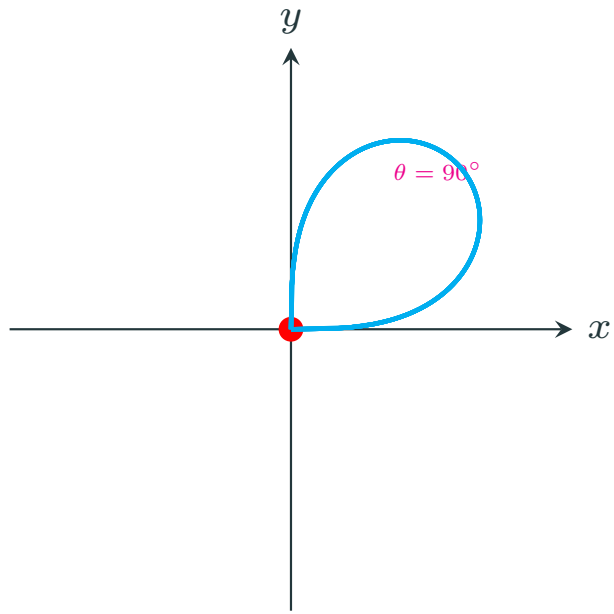

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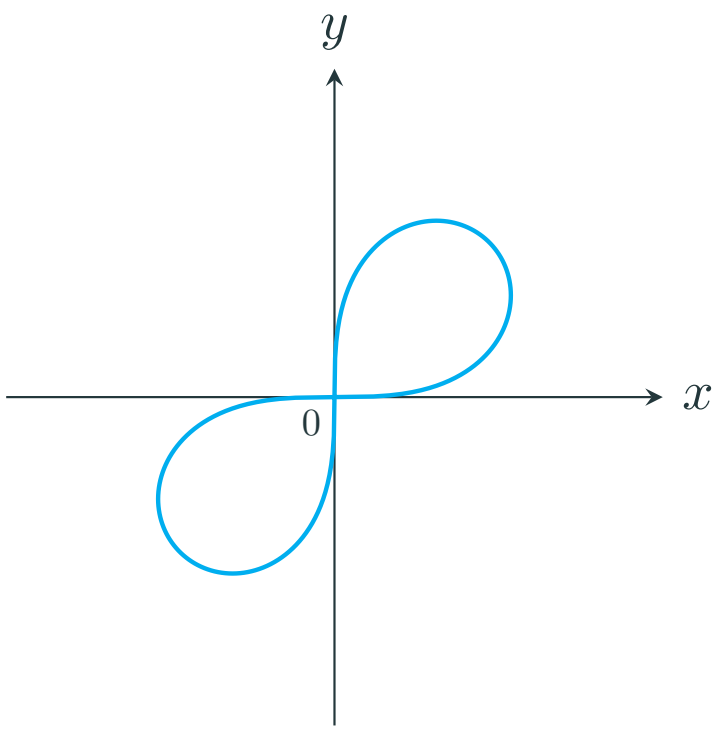

Graphing Polar Equations

Rose Curves

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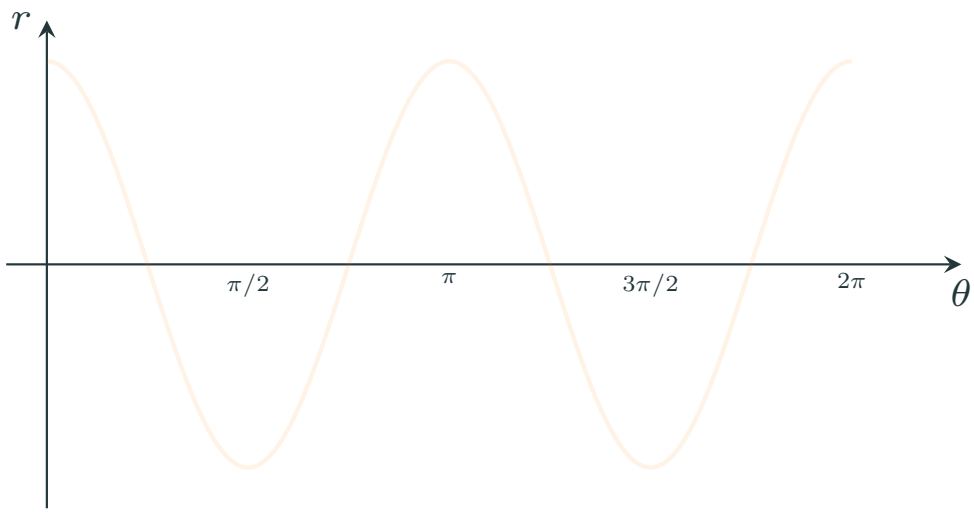
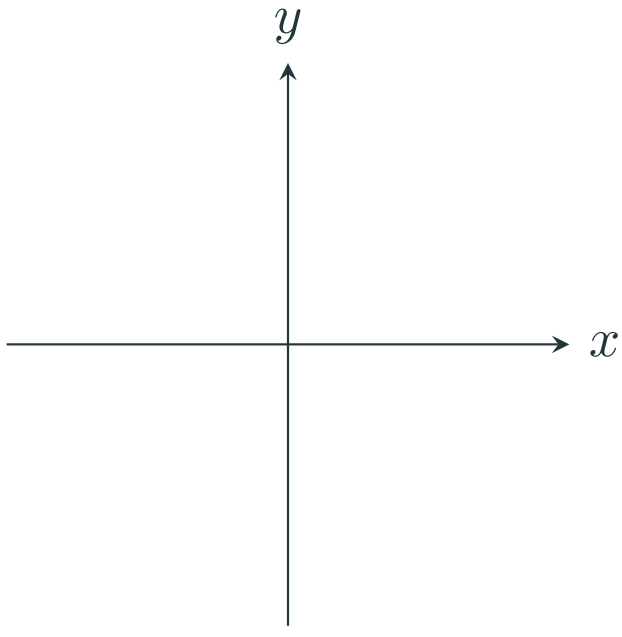
Cartesian $r\theta$-plane guide





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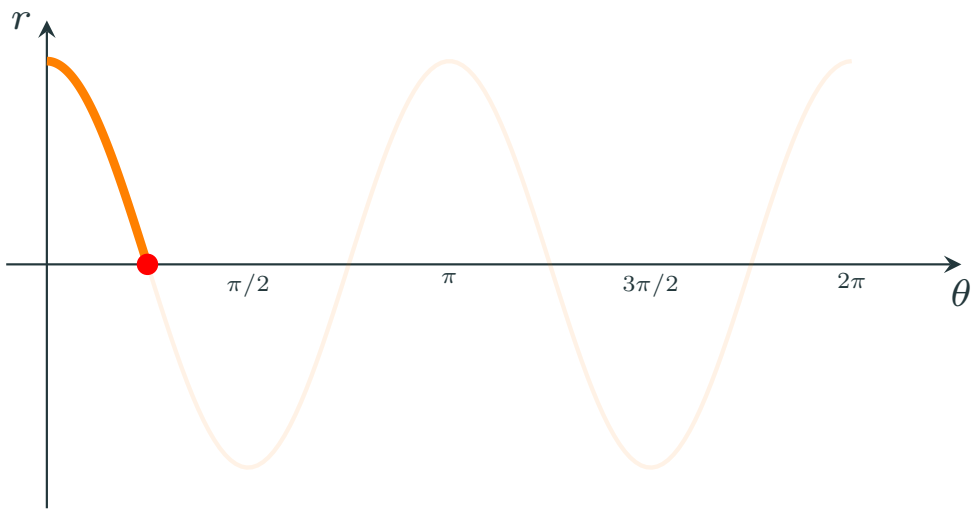

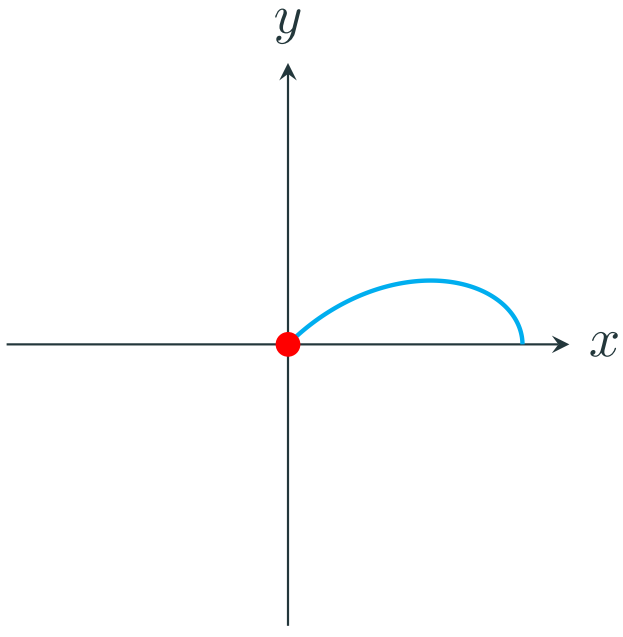

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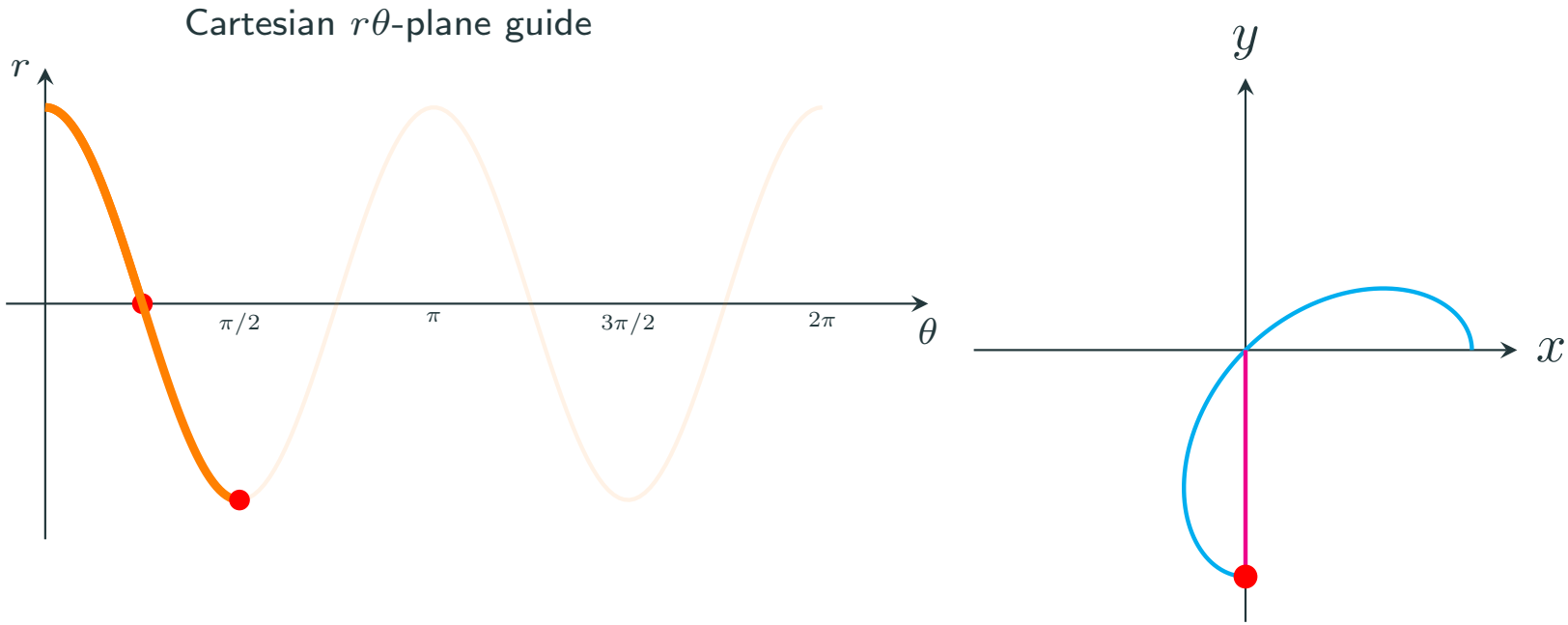

- **At $\theta = \pi$:** The trace completes the upper petals and half of the side petals.

Graphing Polar Equations

Rose Curves

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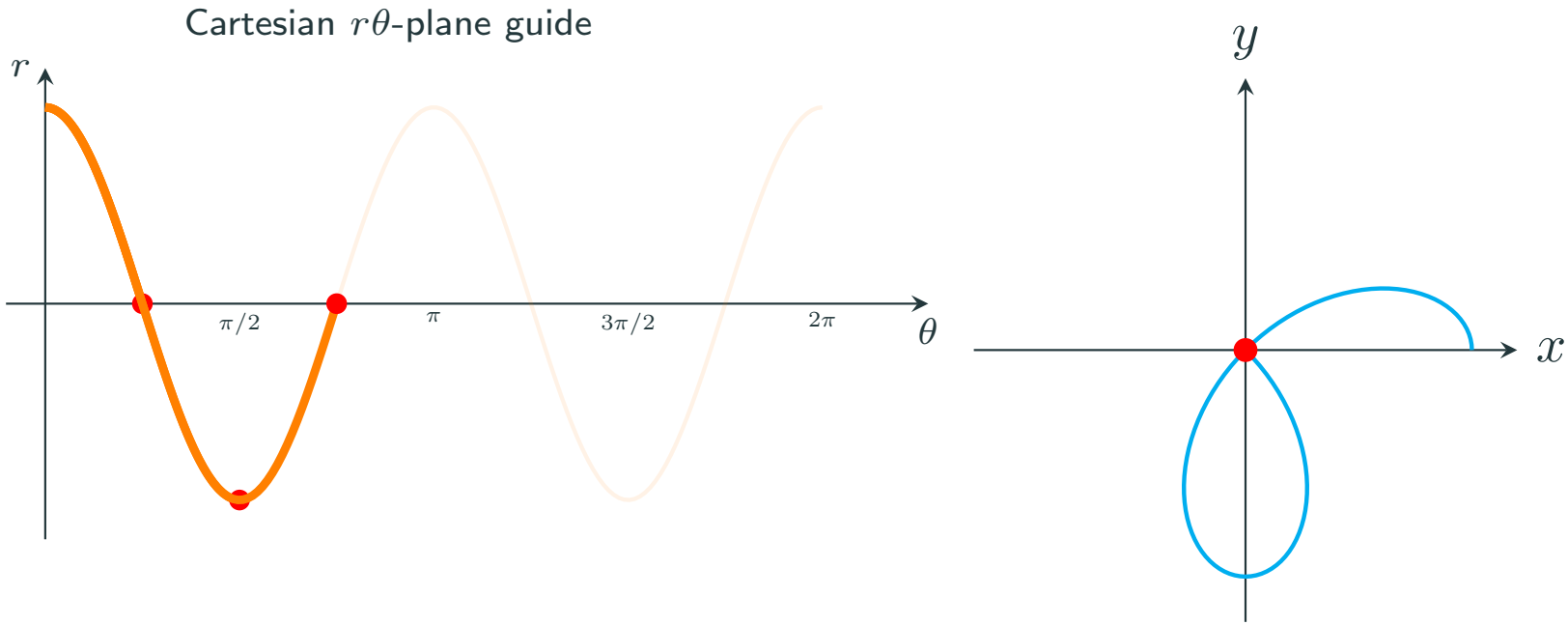

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Graphing Polar Equations

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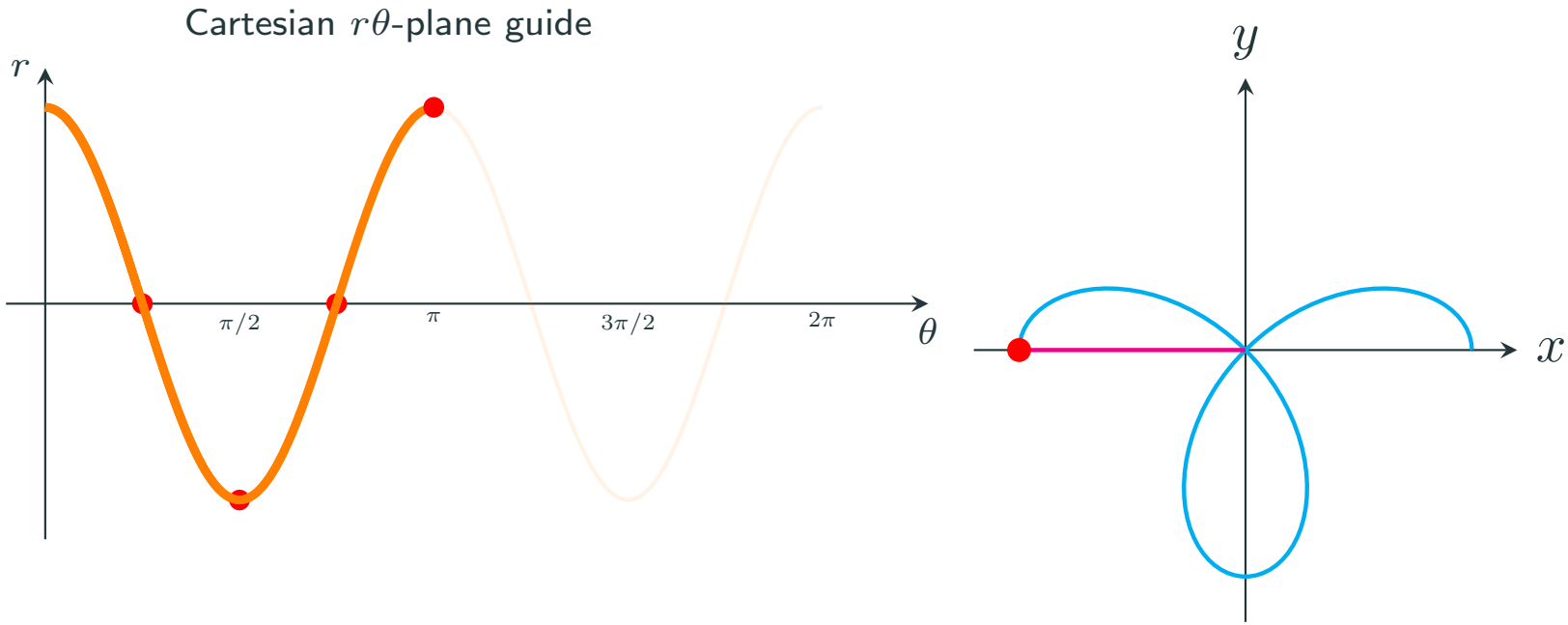

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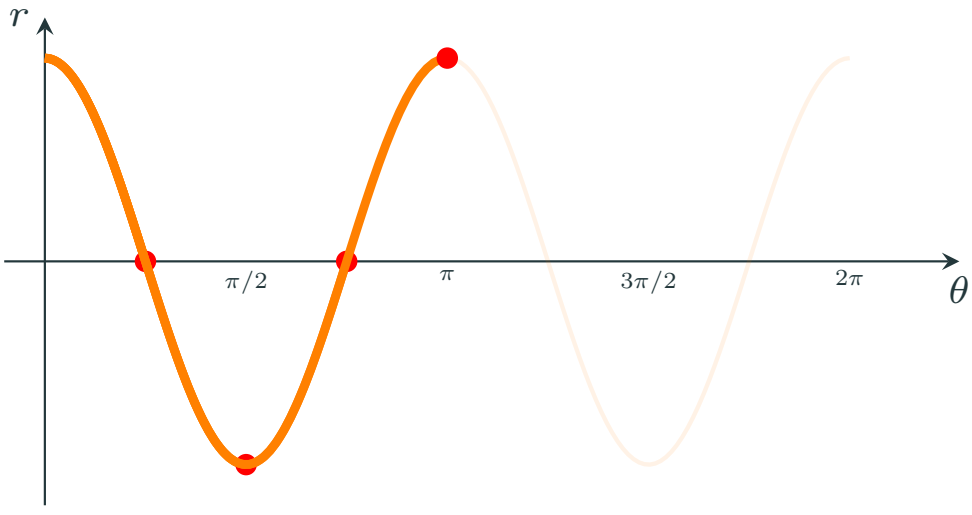

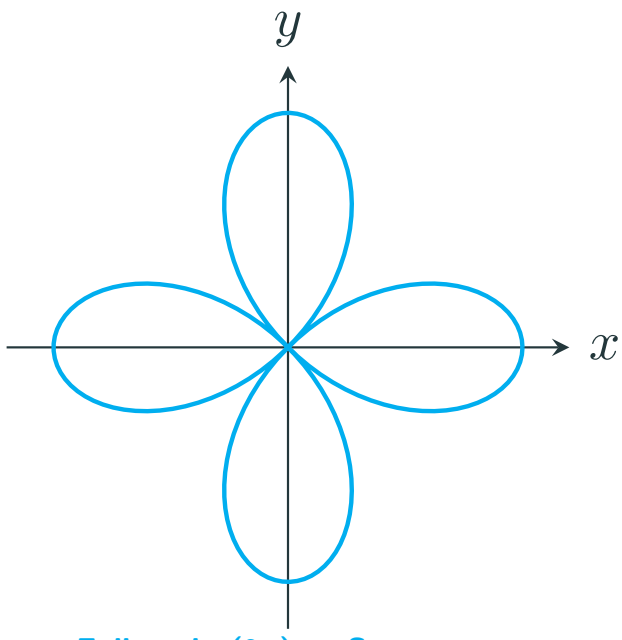

Full cycle (2π) or Symmetry

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Graphing Polar Equations

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EXAMPLE:
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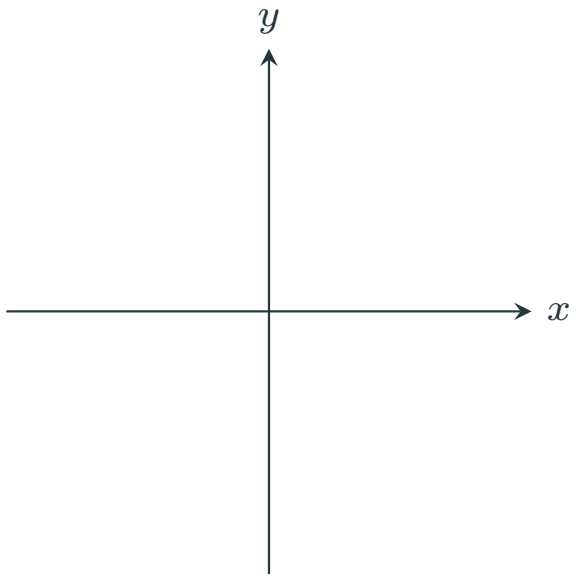
Cartesian $r\theta$-plane guide

Polar Graph (3 Petals)



- **Petal Formation:** Each 60° interval in θ completes one petal.

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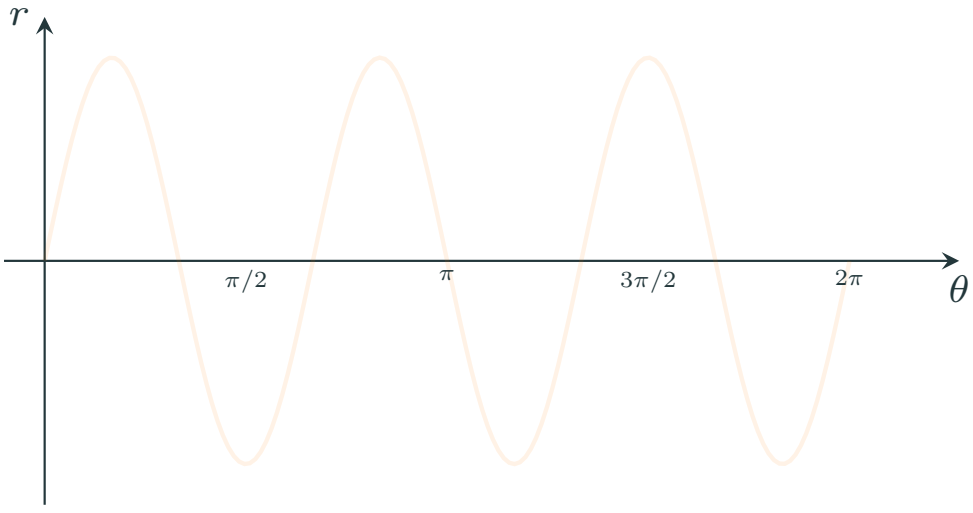

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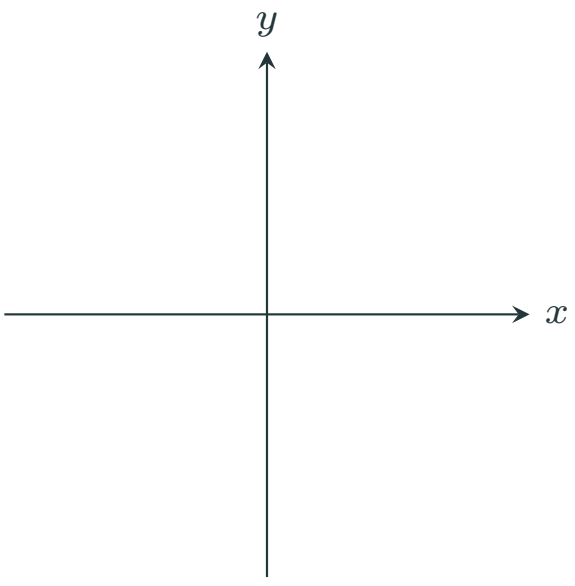

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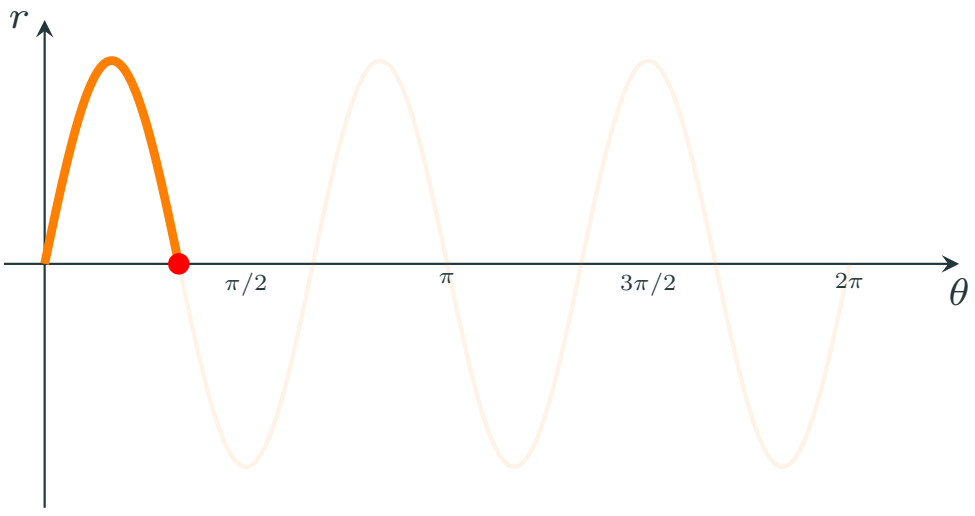

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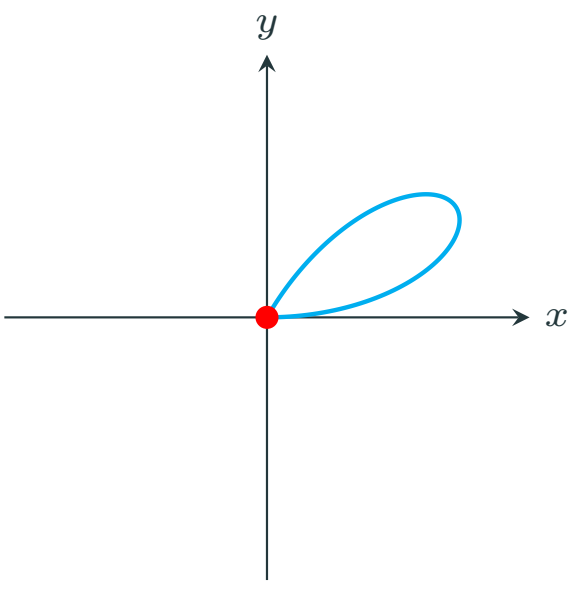

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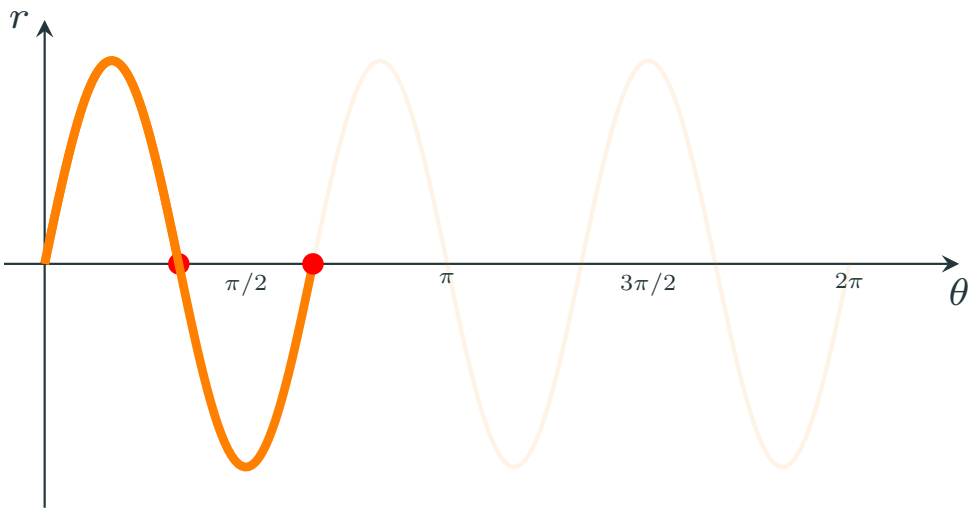

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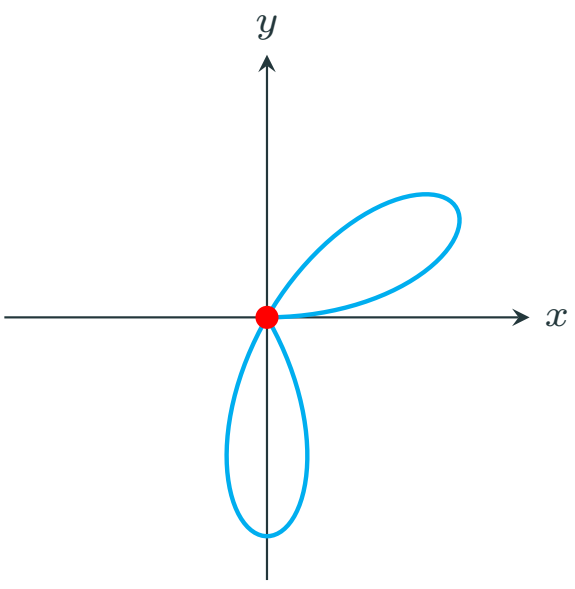

- **Key Observation:** For $n = 3$ (odd), the graph is fully traced once as θ goes from 0 to π.

Graphing Polar Equations

Rose Curves

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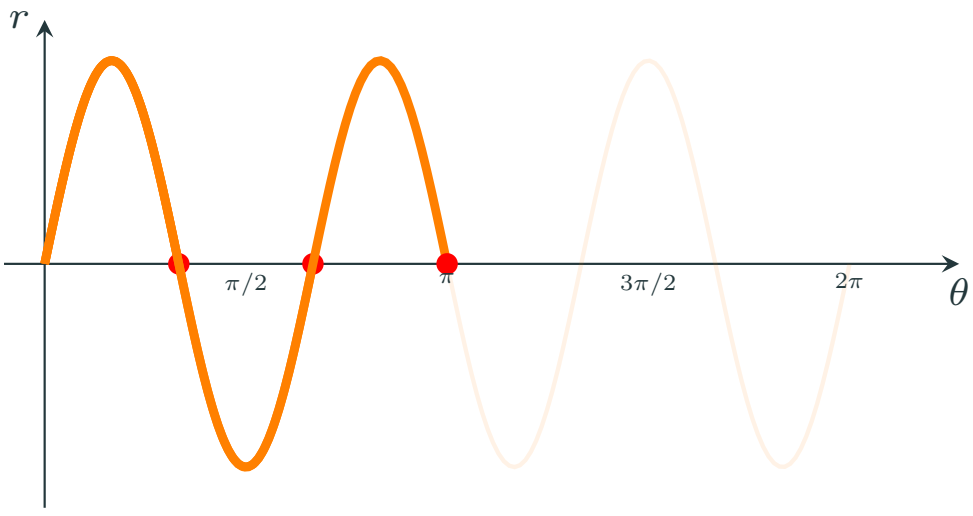
Cartesian $r\theta$-plane guide



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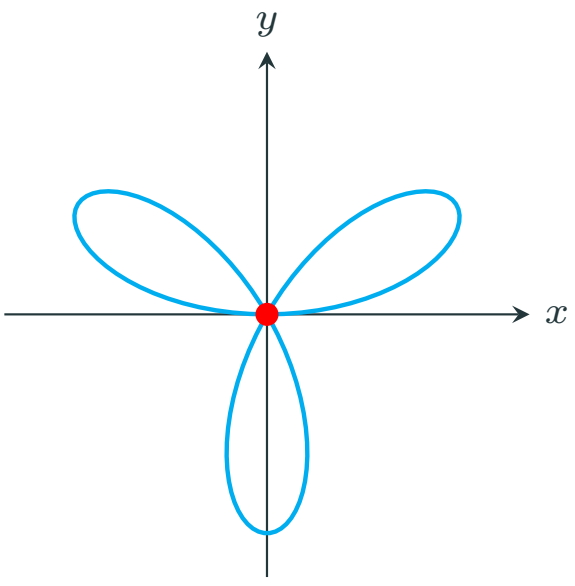

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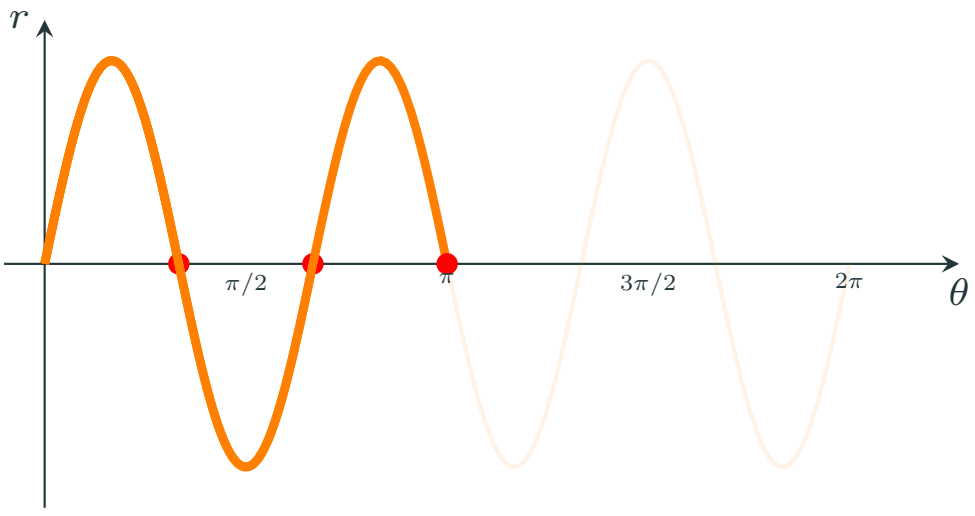
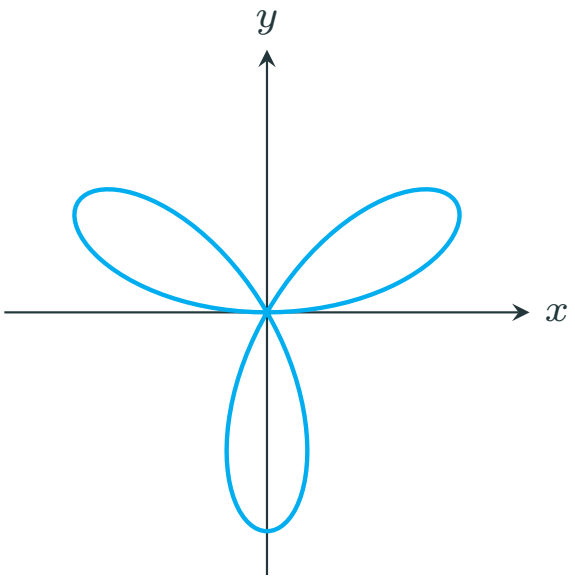


Polar Graph (3 Petals)



Full 3-petal rose (at $\theta = \pi$)

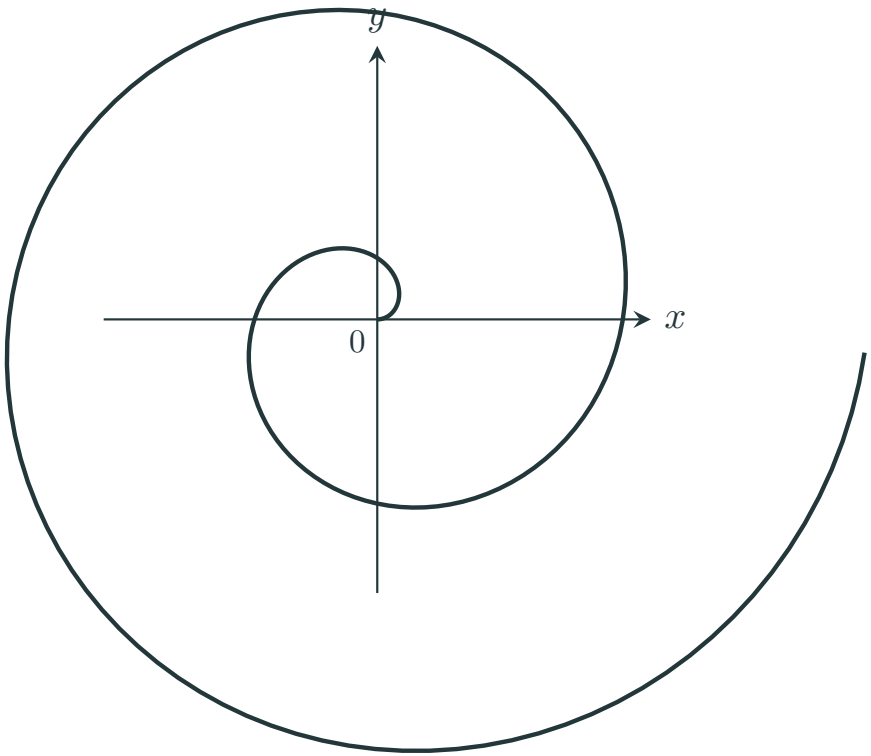
- **Key Observation:** For $n = 3$ (odd), the graph is fully traced once as θ goes from 0 to π.

Graphing Polar Equations

Polar Spirals

EXAMPLE: Sketch the polar graphs of two common spirals.

(a) The equiangular spiral $r = \theta$

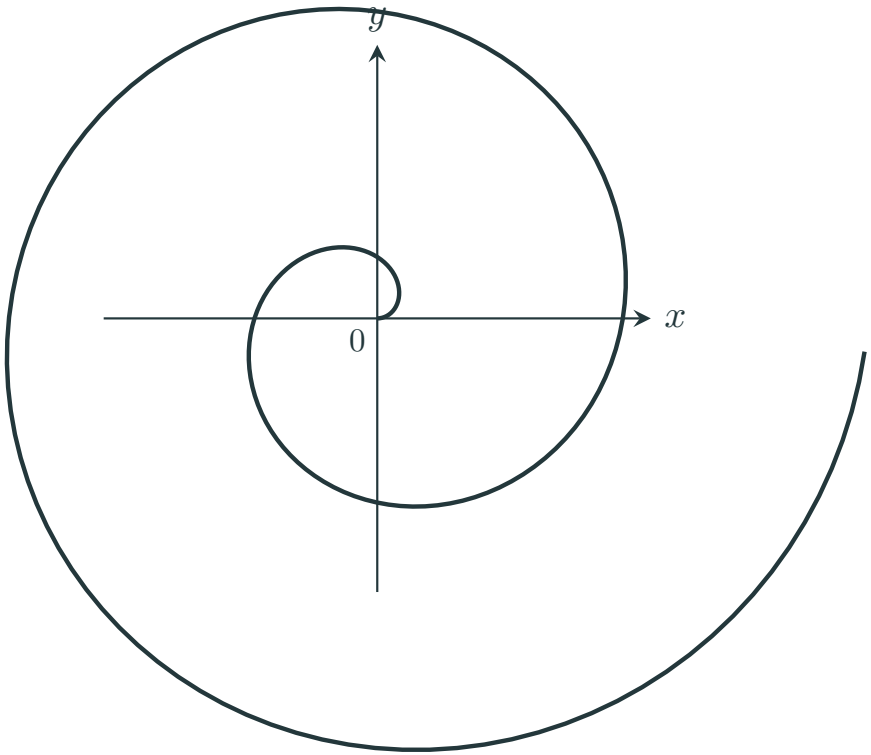


(b) The exponential spiral

$$r = e^{-\theta/3}$$

EXAMPLE: Sketch the polar graphs of two common spirals.

(a) The equiangular spiral $r = \theta$



(b) The exponential spiral $r = e^{-\theta/3}$